Learning and Advancing by Investing:

An Engine for Technological Change and

Endogenous Economic Growth

by

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Declaration

In compliance with the rules of Degree of Doctor of Philosophy of Australian National University it is affirmed that, except where otherwise stated, the work contained in this thesis is my own.

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Abstract

The emergence of new growth theory in the mid-1980s prompts new interest in explaining the long-run growth performances of an economy. Despite all the differences, both traditional and many endogenous growth models consider technological progress as an engine for growth. The difference is how technological progress evolves and which factors can influence the pace of this progress.

The research undertaken in this thesis represents an attempt to construct an endogenous growth model using a specification of the technological progress function, derived from the micro-economic literature. The underlying premise is that by investing, people not only build up the physical capital stock but also build up new knowledge through the learning inherent in the process of investing. Therefore, by investing, people learn and advance to a new level, and this represents an important engine for technological change and endogenous economic growth. A special technology-shock generating function which relates the rate of investment (considered as a technology-enhancing factor) to the increase in the productivity is constructed, using a modified vintage capital model. Under some conditions regarding the shape of the technology-shock generating function, multiple balanced growth paths may exist. Therefore, the economy may end up in different long-run growth paths, depending on time-preference and learning ability among other factors. A surprising result is that although an improvement in learning ability is welfare enhancing, it effects on long-run growth are ambiguous: the economy may grow faster or slower, depending on which balanced growth path is preferred.

Finally, an application of the basis model for an open, developing economy is considered. Emphasis, in this model, is placed on the importance of imported capital goods which are considered as a technology-enhancing factor. The policy implications drawn from this model are obvious: any policy measure which restrict the availability of more efficient, 'superior', foreign inputs not only will hurt the economy in terms of welfare and temporary growth performance, but is potentially damaging for long-run growth.

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CHAPTER 1

INTRODUCTION

The behaviour of economies in the long-run is of great interest to economists and has its roots in classical economics. After a relatively dormant period during the Keynesian revolution, interest in long-run growth theory revived after the Second World War in response to the pressing need for the reconstruction of Europe and the emergence of new nations. The theoretical foundation of the theory was laid by Solow and Swan in their seminal works in the 1950s. The neoclassical growth theory soon become dominant in growth analysis and held this position for a long time. However, this traditional growth theory turned out to be a big disappointment for practitioners and policy-advisers because of its policy-neutral implications. In the neoclassical growth theory, the only factor which can help an economy to maintain a sustainable growth rate in long-run is technological progress. But this important factor for growth is totally exogenous to the model. Therefore, the economy will eventually end up in a 'steady-state' with exogenous growth in the long-run. Any policy measure, at best can only have a temporary effect on growth but could not do anything to change the long-run growth rate.

However, there is a growing body of evidence supporting the argument that policy is far from growth-neutral and that the long-run growth rate does depend on what people and governments do. Levine and Renelt (1992) show that over 50 variables have been found to be significantly correlated to growth in at least one regression. Although there are some reservations about the significance of these regression results, there is need for a new theoretical framework that can provide a better understanding of the long-run growth process and is more consistent with emerging stylized facts which favour the policy-dependent growth hypothesis.

This long-awaited new growth theory emerged in the mid-1980s with seminal works of Romer (1986) and Lucas (1988), where a positive long-run growth rate is generated by increasing returns, spillover effects and externalities resulting from the human capital or knowledge creation process. Since then, a vast body of literature on endogenous growth models has appeared, prompting both intellectual appeal and empirical interest.

Despite all the differences, both traditional and many endogenous growth models consider technological progress as an engine for growth. The difference is how technological progress evolves and which factors can influence the pace of this progress. At the micro level, technological progress has been studied through the process of inventions, innovations, imitations and technological diffusion. At the aggregate level, the link between technological progress and economic activity is captured in the technological progress function used earlier by Kaldor (1957,1961), Arrow (1962), Sheshenski (1967), Eltis (1973, 1993) and many others. New growth models use this specification in different ways: explicitly in some but implicitly in others.

The research undertaken in this thesis represents an attempt to construct an endogenous growth model using this specification. The underlying premise is that by investing, people not only build up the physical capital stock but also build up new knowledge through the learning inherent in the process of investing. Therefore, by investing, people learn and advance to a new level, and this represents an important engine for technological change and endogenous economic growth.

The plan of the thesis is as follows:

Chapter 2 gives an overview of the recent developments in growth theory. It points out the relevance and some drawbacks of the traditional growth theory, initiated by Solow and Swan in the 1950s. Five classes of new growth models are briefly considered. The Chapter also gives an assessment of the new growth theory and its policy implications as well as summarizing recent empirical work on growth. From the methodological viewpoint, testing new growth models should be carried out on a new basis, incorporating underlying assumptions of the model, rather than dealing with a simple reduced equation.

Chapter 3 considers the effects of investment on technological change and, hence, on long-run growth via the learning effects. It is argued that while investment aims directly at building the capital stock, it also can raise the effectiveness of the capital due to the technological diffusion and the learning process involved. The idea was initially developed by Arrow (1961) and has subsequently been used in many works, especially in some recent endogenous growth models . The model proposed in this chapter continues this tradition. However, unlike the preceding models, it relates the investment activity with a lasting productivity shock. The technology-shock generating function which relates the rate of investment (considered as a technology-enhancing factor) to the increase in the productivity is assumed to be non-linear and have a logistic or an S-shaped form. Using a modified vintage model shows that the S-shaped assumption can be economically justified. In the presence of this important property of the technology-shock generating function, the economy may have multiple balanced growth paths. The growth pattern of the economy, therefore, may well be endogenous: different savings regimes, for example, may lead to different accumulation paths with different *long-run* growth rates.

Chapter 4 can be considered as a natural extension of Chapter 3. An intertemporal dynamic optimization problem is considered, so the saving rate, and the rate of investment are endogenized. The model, therefore, is dealing with growth and welfare in a closed-economy context. The exceptional properties of the model such as the non-convexity of the technology-shock generating function, the discontinuity of the induced effective capital function, means that the optimal solution is at one of the two balanced growth paths. Given a possible trade-off between current consumption and the long-run growth rate, an impatient society, which is characterized by a higher discount rate would probably prefer the lower growth path, while a more forward-looking society with lower time preference would choose the higher growth path. The long-run growth rate, therefore , is a step function of the time preference. A comparative static analysis is also carried out in order to examine the role of learning ability in the growth process. It is shown that, while an increase in learning ability is welfare enhancing, it does not also lead to an improvement in the growth performance. On the contrary, the growth rate may decline if the economy is so impatient that an increase in learning ability translate into higher current consumption and lower investment, resulting in higher welfare even though the new long-run growth rate is smaller.

Finally, Chapter 5 presents an endogenous growth model for an open, developing economy. Emphasis is placed on the importance of imported capital goods for the growth performance in the context of a developing country. The relevance of imported capital goods for growth is well documented, and the model aims to provide a possible justification for this. In this model, the economy is engaged in trade with the rest of the world by importing capital goods from outside, because these capital goods are essential and more effective for the home-country. The import is possible either by borrowing or buying directly at the expenses of domestic consumption and domestic accumulation. Foreign inputs are assumed, for simplicity, to be the only source of technological change and the change in productivity is affected by the rate of growth of imported capital goods.

The main results are similar to those in Chapter 4. In the presence of multiple balanced growth paths, time preference, learning ability and the degree of technological novelty and effectiveness embodied in foreign inputs are important factors determining the long-run growth rate of an economy. The policy implications drawn from this model are obvious: any policy measure which restrict the availability of more efficient, 'superior', foreign inputs not only will hurt the economy in terms of welfare and temporary growth performance, but is potentially damaging for long-run growth.

CHAPTER 2

ENDOGENOUS GROWTH THEORY: AN OVERVIEW

2.1 Introduction

Growth theory has the objective to examine and explain the long-run or potential pattern of changes in the rate of growth of the main macroeconomic variables. These variables are national income, the size and compositions of the labour force and capital stocks, GDP per capita, capital per worker, real investment and consumption as well as the real wage rate and the profit rate (factor shares in total income). These indicators are very important for an assessment of a country's economic performance and have been a subject of interest in economic theory since the time of classical economists and non-mainstream economists. In the 1920s, Ramsey brought growth theory to a new stage, when he used an optimizing model to address the question ' how much should a nation save?'. Then came the era of the Keysian revolution in economics. Overshadowed by the famous Keysian statement 'In the long-run, we are all dead', both the economic profession and policy-makers concentrated their efforts on static general equilibrium approaches. Short-term and medium-term expansion of the economy were the focus of analysis by different schools and approaches, but the determinants of long-term growth were left almost unexplained. It was not until the 1950s, with the work of Harrod, Domar, Solow and Swan, that growth theory was back again and destined to play a dominant role for many decades. The radical propositions of this theory on total exogeneity and policy-neutrality of long-run growth, resulted in a dissatisfaction among growth theorists and practitioners. Partly as a reaction, the economics of developing countries formed its own branch to offer the direct policy advice that growth theory could not. Some growth theorists kept working on different aspects of the theory, making departures from the traditional approach, As a result, by the middle of the 1980s, a new wave of the growth theory had emerged, named ' the endogenous growth theory'

2.2 Kaldor stylized growth facts

In 1958, Kaldor suggested six " stylized facts" as starting points for the construction of theoretical models of growth. These facts refer to the long-term regularities in the relationships that seem to appear in most industrial countries, between growth rates of output and capital and labour inputs and between factor prices and relative income shares. These are the facts or rough empirical observations that a growth model must explain to be convincing and with which the model must be consistent. Following Romer (1989a), these facts can be listed as follows:

1. Output per worker shows continuing growth 'with no tendency for a falling rate of growth of productivity'
2. Capital per worker shows continuing growth
3. The rate of return on capital is steady
4. The capital-output ratio is steady
5. Labour and capital receive constant shares of total income
6. There are wide differences in the rate of growth of productivity across countries.

The statistical data on economic performance around the world strongly support all these Kaldor stylized facts and are very well reported in the vast literature.

Romer (1989a), for example, using the Summers-Heston table and Maddison work, shows that over the last century, the output per man-hour increased by 12 times in the United States, 26 times in Japan and 5 times in Australia. He also pointed out that the output to capital ratio in developed economies is fairly steady and lies between 0.3 to 0.4. Overall, these stylized facts are well observed and can serve as a practical criterion to establish the suitability of any growth model.

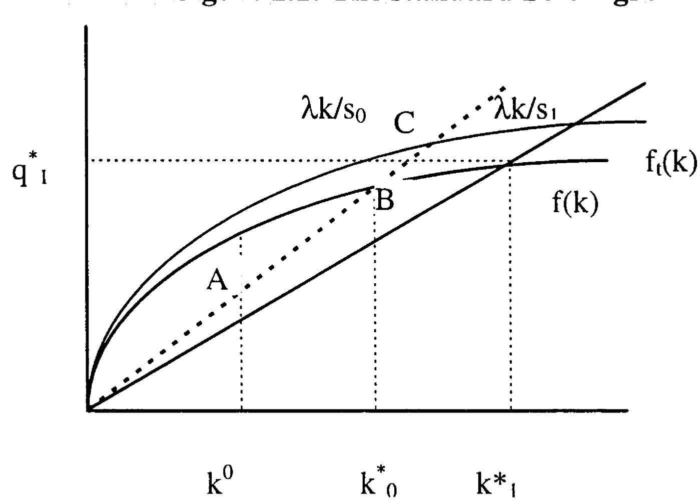
2.3 The traditional neoclassical growth theory: Solow-Swan model

The first successful formalized growth model was initially developed independently by Solow and Swan in 1956. The economy, in this model consists of a single firm using two factors: labour and capital to produce a single output. The labour force is assumed to grow at a constant exogenous rate λ. So if at the starting point, the economy is endowed with L_0 units of labour force, then at moment t the total labour force available is $L_t = L_0 e^{\lambda t}$. The output Q is a function of inputs that exihibits constant returns to scale, substitutability between capital and labour, and most importantly, diminishing marginal productivities. That means: firstly, if both inputs are changed proportionately, then the output will also change by the same proportion (constant returns to scale property); secondly, if one of the inputs falls, another input must increase if the total input is to be maintained at the previous level (factor substitutability): and finally, other things being equal, as long as one factor increases, its marginal contribution to the total output will decline and approach zero. The last property of the production function, in the growth literature, is often referred to as the Inada condition.

In the first version of the growth model proposed by Solow in 1956, the dynamics of the economy were considered under another additional assumption, that is investment and saving are a fixed fraction of output. Under these conditions, the growth pattern of the main macroeconomic variables in per capita terms can be graphically shown in Figure 2.1.

Initially, the economy is at point A, with capital per head equal to k_0. The actual investment per person, defined by saving ratio times the output per capita is higher than the level required for maintaining the actual level of capital per head. Therefore, capital per head is rising. The economy moves upward, along the production function curve *f(k),* away from the initial point A. This process continues until the economy reaches point B. At this point, the economy reaches its equilibrium point or its steady-state, and no further increase in the income and capital stock per capita occurs. The economy stops growing in the long-term.

Figure 2.1: The standard Solow growth model.



This standard textbook exposition of the Solow growth model can be used to analyse the effects of changes of some exogenous variables on the level of income and capital per head. For example, a shift upward of the production function due to an exogenous technological change or due to an increase in non-reproducible resource endowments of the economy, will eventually increase the levels of both income and capital per head. However, eventually, the economy will move along the new production curve and approach another stationary position with no further growth in income and capital. The economy can only keep going if there is a sustainable shift in the production function caused by exogenous technological change, totally independent of the inside economic activities. This represents an important escape clause for the neoclassical growth model in explaining the long-run growth rate, but implications for policies are very crucial: all sources of long-run growth are totally exogenous and cannot be explained inside this model, the growth pattern of the economy is totally policy-neutral in the long-term.

The long-run growth implications of the Solow model contain both pessimistic and optimistic elements. They are pessimistic, because, according to the model, the economy will eventually end up with no growth in the long-run if there is no change in technology. The optimistic element of the traditional growth theory consists of its prediction about the potential convergence of all economies with the same technology and the saving rate to the same level of income per capita.

The Solow model maintained its dominant place in growth theory during the 1950s and the 1970s. It, in fact, can explain at least five of six Kaldor stylized facts. Most importantly, the model was considered as a very good device for analysing the growth pattern at least in the short and medium-term. The methodology, underlying the model became a basis for growth accounting practices for much empirical work (Maddison, Denison). The conclusion of the Solow model that the rate of economic growth in the long run is equal to the rate of exogenous technical change became very popular and widely accepted by many economists. Even in the early 1980s, this dominant place held by the neoclassical growth theory was so strong that some even declared " these [Solowian] propositions are obvious, almost trivial, and the source of confusion on this issue ... is not ultimately empirical, or even economic, but linguistic." (Usher, 1980, 260-1)

However, from the outset, the Solow-Swan model already exposed some weakness that made many economists fell uneasy. First of all, from the theoretical point of view, the existence of a steady state, where all variables cease to grow, is very nice for the theory itself, but not for the task the theory is called upon to do. Solow realised this problem and he confessed: ' My general conclusion is that the steady state is not a bad place for the theory to start , but may be a dangerous place for it to end' (Solow , 1970). With this implication of the neoclassical model, according to Scott (1989), we are left with the paradoxical conclusion that this theory of growth has nothing to say about what determines the equilibrium rate of growth. In this sense, such a model can be labelled a 'no-growth model' (Dowrick, 1993).

From a practical point of view, the Solow model is unable to give a comprehensive explanation of some of the most important stylized facts concerning recent economic growth. As Dowrick (1993) put it, these facts are:

i. the exceptionally high rates of growth of the 1950s and 1960s. This period was called 'a Golden Age' for Japan and Europe, where the average annual rate of growth of GDP was 9.1 per cent for Japan, and 4.8 per cent for most European economies

ii. the subsequent worldwide slowdown, especially during the 1973-1989 period (Japan- 3.9 per cent, Europe-2.0 per cent, (Boltho and Holtham, 1992)

iii. the tremendous variance in growth rates across countries, especially amongst the middle-income economies. An excellent example of this growth variation can be found in the growth performance of Latin America and East Asian countries.

iv. the apparent convergence of productivity and living standards amongst the most advanced economies (in accord with the predictions of the Solow-Swan model) but divergence for less developed economies, despite the rapid expansion of trade and capital mobility.

To this list can be added another fact concerning capital movement between countries. The Solow model would suggest that if two countries have the same technology, then under perfect mobility conditions, capital would move from a country with a higher capital-output ratio to a country where this ratio is lower, because of differences in the marginal returns to capital. No evidence supports this prediction. On the contrary, Mulligan and Sala-i-Martin (1992) found that the pattern of capital movement is somehow different and cannot be fitted into the neoclassical growth model. They found that capital actually moves from countries with *lower growth rates* into countries with *higher* growth rates, rather than from rich to poor countries as implied by the neoclassical model.

Table 2.1. Effects of Some Factors on Per Capita Output Growth.

Parameter	Observed Effects on Per Capita Output Growth	Solow-Swan Predictions	
		Short-run	Long-run
Domestic Saving Ratio.	Positive	Positive	No
Government Expenditure Share on:		NA	NA
a)Human Capital	Positive	Positive	No
b)Consumption	*Negative*	NA	NA
Growth Rate of Export Volume	Positive	NA	NA
Level of Tariff	*Negative*	NA	NA
Financial Development [a)]	Positive	NA	NA
Human Capital [b)]	Positive	Positive	No
R&D	Positive	NA	NA
Foreign Direct Investment	Positive	NA	NA

Note : a)measured by the ratio of liquid liabilities to GDP(in Robert G. King and Ross Levine. Journal of Monetary Economics, 1993

b) measured by the Barro-Lee index: secondary school enrolment ratio

NA : not applicable

Source: Ichiro Otani and Delano Villanueva (1990), Robert G. King and Ross Levine. (1993), Fisher (1993), Dowrick S.,(1992), Blomstrom M., Lipsey R. E. and Zejan (1992), Barro R. (1991), Lee (1993), Coe D.T and Moghadam (1993), Easterly W. and Rebelo, S.(1993).

The most disappointing feature of the Solow model for many economists-practitioners is its policy-neutral conclusion. As Romer put it: 'From the point of view of policy advice, growth model has little to offer. In models, with exogenous technological change and exogenous population growth, it never really mattered what the government did', Romer (1989a). Yet, there is an enormous amount of empirical work showing that the GDP per capita growth rate, in fact, is affected by a whole set of factors, many of which are not policy-neutral, (Table 2.1). According to Levine and Renelt (1992), over 50 variables have been found to be significantly correlated to growth in at least one regression. Of course, the conclusions from cross-section growth regressions should be carefully interpreted because, as Dowrick (1993) and Fagerberg (1994) pointed out, many statistical issues concerning the exogeneity and interrelatedness (or interdependency) of explanatory variable cannot be satisfactorily solved in these cross-section regressions. Nevertheless, there is a wide consensus among growth theorists and practitioners that the growth rate is far from policy-neutral. This state of affairs led to the emergence of a new wave of growth theory which become known as ' new growth theory'.

2.4 Recent endogenous growth theory

Partly as a reaction to omissions and deficiencies in the traditional growth model and its limited practical value, there was an emergence of a new wave of growth theory in the 1980s. The phrase 'endogenous growth' has become very popular and fashionable in the economic lexicon during the last decade after the appearance of two articles published by Romer in 1986 and Lucas in 1988. The endogenous growth theory embraces a diverse body of theoretical and empirical work which 'distinguishes itself from neoclassical growth by emphasizing that economic growth is an endogenous outcome of an economic system, not the result of forces that impinge from outside' (Romer, 1994). These endogenous models can be classified into the following five prototypes, (Sala-i-Martin, 1990):

2.4.1. Convex (linear) growth model

This kind of model was first proposed by Rebelo in 1990 and also by Jones and Manuelli (1990) in a more complicated form. The model makes a significant departure from the Solow model by assuming that the production function is a linear function of capital i.e. Y = AK. For this reason, this model is often referred to as the 'AK model'. The marginal productivity of capital in this case is equal to its average productivity and is constant (something similar to the Harrod-Domar assumption). The law of diminishing returns to factors is no longer valid, so the economy can have a sustainable, positive growth path under some well specified conditions.

This was the first neoclassical model that relates the long-run growth rate of a country to its saving ratio. In fact, the model states that the growth rate depends on its saving rate and on how productive is its technology: a country with a higher saving ratio and more productive technology will enjoy a higher long-term growth rate.

The problem with this model resides in its extreme assumption on the constancy of the marginal productivity of capital. If this capital is the usual physical capital in the traditional growth model, then the production function would exhibit increasing returns to scale, should other factors like labour be included. In this case, a market competitive equilibrium could not exist. Therefore, capital, here is understood as capital in a broad sense, including not only physical but also human capital, as well as stock of knowledge and financial capital.(Sala-i-Martin, 1990). However, this abstract notion of capital limits the practical value of the model.

Rebelo (1991) later modified this model by considering an economy, which consists of two sectors: consumption and investment. The consumption sector produces consumption goods using two inputs: physical capital, in the usual sense, and labour and has the usual Cobb-Douglas production function. The investment sector produces investment goods using capital as the only input, and has a linear production function. The long-run growth result is similar to that in the first version.

2.4.2. The Barro model of public spending

This is a growth model which tries to link growth to fiscal variables: taxation and government spending. It is assumed that government imposes an income tax at a flat rate and uses the tax revenue to provide public services. These services, according to Barro in his first version, are provided to private users without charge but are rival and excludable and serve as an input to private production. Therefore, the aggregate production has the following form:

$y = g^{1-\alpha}k^{\alpha}$ ($0< \alpha <1$).

The growth pattern derived from this model is similar to the Rebelo model. Given initial values of capital stock, government spending and income tax rate, the economy is always in a position of steady-state growth in which all quantities grow at the same rate. This rate is positively related to the size of government spending and negatively related to the tax rate. The final outcome depends on the relative sizes of these two effects.

The role of the government in maintaining sustainable growth is clear. Every time private individuals decide to save one unit of consumption and purchase one unit of capital with it , the capital stock would rise by one unit, and the marginal productivity of capital would fall. However, with the government action of providing free public input, this fall in productivity can be avoided and growth can be maintained at the same rate forever.

This model of public spending suffers the same increasing returns problem that the first model faces.

2.4. 3. Learning by Doing, externalities and increasing returns, (Romer-Arrow-Sheshenski)

The idea that technological change is not totally exogenous is not new. Some unorthodox growth theorists in the 1950s and 1960s had already mentioned a possible relationship between technological change and the state of the economy. Kaldor (1957), for example, argued that:

> A society where technical change and adaptation proceed slowly, where producers are reluctant to abandon traditional methods and adopt new techniques, is necessarily one where the rate of capital accumulation is small. The converse of this proposition is also true; the rate at which a society can absorb and exploit new techniques is limited by its ability to accumulate capital.
>
> (Kaldor, 1957, p595)

Arrow (1962) advanced the hypothesis that technological change can be associated with experience. He used cumulative gross investment as an index of experience, and argued that the change in the stock of knowledge will affect productivity. So the state of technology can be considered as a function of total capital stock, (K). Sheshinski (1967) clarified Arrow' s idea and proposed an aggregate production function of the form for an individual firm: y = F (k, A(K)l). Therefore, there exists a positive externality for a firm due to an increase of the capital stock of other firms. The aggregate production function exhibits increasing returns. The rate of technological change now becomes endogenous, but the growth rate remains exogenous in the long-term.

This approach was revised by Romer (1986) in his first model. The difference is that the technological change now is no longer a side-effect of the capital accumulation process but is a product of a new specialised sector producing new knowledge intentionally. The increase in the stock of knowledge is a function of the new investment to capital ratio, and the overall stock of knowledge has a positive external effect on the productivity of an individual firm and results in the aggregate production function having increasing returns. Under some additional assumptions on the production function and the knowledge creation function , the economy can grow at some positive, sustainable and even explosive rate.

2.4.4. Human capital accumulation (Lucas, 1988)

This model focuses on the effect of human capital accumulation on the long-term growth rate. The economy consists of two sectors: one produces consumption goods, using physical capital K and human capital h , and the other sector specialises in the development of human capital . An individual devotes a fraction of his human capital, u to the production of consumption goods, and $1-u$ to the process of human capital accumulation. The average stock of accumulated human capital has an external effect on the productivity of the consumption goods sector and grows at a constant rate, given the fixed fraction of human capital devoted to this accumulation process. Formally, the model can be written in the following form:

$$Y = AK^{\beta} (uhL)^{1-\beta} h_a^{\varphi}$$
$$\dot{K} = Y - c$$
$$\dot{h} = \phi h(1-u)$$

Note that the aggregate production function exhibits increasing returns if we take into account all factors, and the sector producing knowledge has constant returns. This crucial assumption makes the sector a driving force for the growth of the whole economy.

Under these assumptions , the growth rate of the economy will be positively related to the productivity in the human capital creation sector. The more productive is this sector (the bigger is the parameter ϕ) the higher will be the long-term growth rate. Furthermore, when the externality is positive, the optimal growth rate for the human capital creation sector (that is the rate which maximises the social welfare function) is always larger than the market rate, because the incentive for the private sector to invest in human capital is lower than the social incentive. The policy implications of the model are quite straightforward: any policy which is aimed at the improvement of quality and quantity of human capital will have a positive effect on the growth rate of the whole economy.

2.4.5. Research and Development (R&D) growth model (Romer, Grossman and Helpman)

This type of endogenous model can be found in the second work of Romer (1989b) as well as in Grossman and Helpman (1990). This is the most developed endogenous growth model, because it provides a sophisticated and comprehensive microeconomic foundation for technological change. The economy in these two cases is characterized by wide specialisation and division of labour. There are many sectors in the economy, producing consumption goods, (with different varieties in the Grossman and Helpman model), or durable production goods (Romer model) and a special R&D sector specialising on the production of new designs for intermediate or final consumption goods. The latter sector has constant returns to labour and increasing returns overall, which allows this sector to grow at some constant rate and become a driving force for the growth of the whole economy.

In the Romer-style model, the growth rate positively relates to the productivity of the research sector and the amount of skilled labour devoted to the research activity. Furthermore, in the presence of externalities induced by products in the research sector, the level of human capital devoted to R&D by the private sector in the market equilibrium state, is lower than that needed for the social optimum. The policy implications of the model, therefore are very clear: in order to have higher growth, it is necessary to maintain a higher level of human capital by developing the education and training system, as well as, undertaking some subsidy measures in support of the research activity.

In summary, the new wave of growth theory during the last decade has taken a significant further step in explaining the driving force for growth. The most common and significant feature of all types of endogenous growth models is to try to demonstrate that technological advancement, and hence, the long-term growth rate, depends on what people do, as Romer (1994) put it. From the methodological point of view, all types of endogenous growth models have stressed the importance of advances in knowledge which lead to improvements in productivity and prevent the marginal productivity of capital from falling to zero when the process of capital accumulation takes place. The difference is that each type of model relies on the idea that only certain *kinds* of investment can do this - R&D expenditure, investment in education, or the like (Scott, 1992). This makes the new growth theory, as a whole, quite fragmented because each type of model focuses on different aspects of factor accumulation, and hence the sources for growth vary in different models. Nevertheless, some policy implications are apparent in all endogenous growth models. The theory calls for the active intervention by the government to foster growth of the driving factor (human capital, education, research, public services, etc) when there is a gap between market and social optima.

Another common feature of most types of endogenous growth models is the assumption of a linear (constant),or increasing returns production function in *at least* one sector. This is a very strict assumption, but it is vital for the model to generate positive long-term growth. It is this strict assumption that makes some economists feel sceptical about the new growth theory. Solow (1994) states:

> 'The idea of endogenous growth so captures the imagination that growth theorists often just insert favourable assumptions in an unearned way; and then when they put in their thumb and pull out the very plum they have inserted, there is a tendency to think that something has been proved'.

Solow (1994, p 53)

Furthermore, this linear (constant returns),or increasing returns assumption may lead to an unrealistic, unbounded growth pattern in some models. It should be noted that the comment made by Solow is , to some extent, true for some, but cannot be applied to all new growth models. Many new growth models like those of Romer and Helpman and Grossman do have a solid and sophisticated microeconomic foundation and the underlying assumptions are well fitted into the neoclassical paradigm

It should be noted that the above classification of new growth models is not unique and cannot exhaust all existing growth literature. Rather, it captures only the main neoclassical growth models where the dominant theme derives from the theory of firm and production in a competitive industry. All above-mentioned models have a production function as their indispensable element. The time paths of the main economic variables such as output, inputs and prices are interpreted as the paths generated by maximising firms (and/or rational consumers) in a moving equilibrium driven by changes in product, demand, factor supply or technological conditions (Nelson and Winter (1974)) . Other things such as organisations of firms, institutional factors and changing of decision rules by firms are set aside as given. These unexploited sources of growth are the focus of attention of many other non-mainstream growth theories such as the evolutionary theory by Nelson, Winter et al., the technological gap theory by Posner (1961), Gomuka (1971), Cornwall (1976) , as well as the theory of specialization and division of labour developed by Ng, Borland and Yang (1991,1993). This research provide many useful insights into explaining growth and represents special interest.

2.5 Empirical evidence

New research on growth theory undoubtedly provides new insights into the sources of long-run growth . The conclusions drawn from different models have important policy implications . But the question is to what extent do the new growth models fit the growth experience? Do these models give a better explanation than the traditional one. To answer these questions we can only test the theory when data are available . But how should a proper test of the new growth theories be designed?

There exists a vast body of empirical work which aims to reveal possible linkages between long-run average growth rates and a variety of economic policy, political and institutional factors . With the emergence of the new growth theory and the availability of a new data set published by Summers and Heston, the interest in this empirical work has become more intensified. The objective of these empirical studies is similar: to find out the possible sources of growth. The list of suggested factors of growth is very lengthy including a wide range of variables characterising fiscal , monetary, and trade policies and many other.

Can we judge the validity of different growth theories by using regression results from the above-mentioned work ? How much confidence should we have in the conclusions?

Ideally, new growth models should be tested *directly* on the grounds of their underlying assumptions such as the presence (or the absence) of increasing returns or externalities, or by testing the validity of some specific structural equations in the models (human capital formation equation in the Lucas model, or the knowledge creation equation in the Romer model, for example). Alternatively, new growth models should be tested by a system of simultaneous *structural equations* rather than by a *singe reduced equation* , because what really makes a new growth model different from another, and notably from the traditional one is its specific structural equation(s). Unfortunately, up to now, there have been very few researchers adopting this approach. Most of the empirical work on growth uses cross-country regression in a single reduced equation. The reliability of these regression results has been questioned by many researchers. Levine and Renelt (1992), for example, argued that although ' there are many econometric specifications in which measures of economic policy are significantly correlated with long-run per capita growth rates...., these relationships are not reliable. A broad array of variables are not robustly correlated with growth: small alterations in the other explanatory variables overturn the past results'. Furthermore, many statistical problems such as the endogeneity and the interrelatedness between explanatory variables cannot be solved within the cross-country regression framework.

However, to some extent, these regression results can be used in testing different predictions drawn from different growth models. In other words, this empirical work can be used to test *indirectly* the new growth theory. The concept which has been widely used for this purpose is known as the convergence hypothesis in its two versions : the absolute convergence and the conditional convergence. According to the traditional growth theory, if countries have the same technology, the same tastes and preferences, then in the long-run, they will end up with the same growth rate . Therefore, during the transitional period towards equilibrium, there is a negative relationship between the initial level of development (measured by the capital-to labour ratio or the initial GDP per capita) and the growth rate: the poorer country tends to grow faster then the richer one, and all countries eventually will have the same level of GDP per capita. This phenomenon (or relationship) is called the absolute convergence. On the other hand, if every country is assumed to have its own technology, taste and preference, then every country may end up with its own long-run steady state. A country will grow faster the farther it is from its own steady-state. This implies that a less developed poor country may grow slower than a developed economy. This is the second ' conditional convergence' predicted by the traditional growth theory.

According to the new growth models, none of these convergence tendencies is necessarily true. Because, the long-run growth rate may be affected by the inside economic activity, depending on what people are doing, there is no reason why the actual growth rate is negatively related to the initial level of development or why the growth rate should slow down at some specific time periods. These fundamental differences between the traditional and new growth theories in the growth predictions are believed to be a *testable* proposition for many empirical researchers. Empirical evidence of growth convergence would serve as solid grounds for accepting the traditional view on growth and therefore, rejecting the new growth models. However, the regression results so far are inconclusive. The growth convergence is accepted in some cases but rejected in some others. Barro and Sala-i-Martin (1995), for example, studied the growth behaviour of the US states since 1880, the prefectures of Japan since 1930 and the regions of eight European countries since 1950 and found that the absolute convergence was the norm for these regional economies: the poor regions of these countries tend to grow faster than the rich ones . One problem is that the speed of convergence is too slow: it would take 25 -35 years to eliminate one-half of an initial gap in per capita, while the predictions of the traditional growth theory would suggest about 11 years, given that the share of capital in total output is about one-third. Furthermore, the convergence failed to be observed when one uses a cross-country data set with developing countries being included, (Barro(1991), Romer (1992), Dowrick, (1992)).

2.6 Conclusion

The major contribution of endogenous growth theory has been to reinvigorate the investigation of the determinants of long-run growth. By recognizing that growth is endogenous and providing theoretical justifications for different sources of growth, the new theory has important policy implications as to how growth can be enhanced. While there is substantial evidence to support the new growth theory in some areas, the overall and adequate testing of new models is far from complete and requires more work to be done. The focus should be concentrated on direct testing of the implications and the underlying assumptions of the new theory.

CHAPTER 3

MODELLING TECHNOLOGICAL CHANGE:

LEARNING AND ADVANCING BY INVESTING.

This chapter explores the effects of investment on technological change. The idea that technology change may be induced by investment activity was initially developed by Arrow through his concept of 'Learning by Doing' which was initiated in the early 1960s and subsequently has been developed in many works, especially in some recent endogenous growth models. A specific form of a technological progress function is constructed, using the idea of vintage capital theory and introducing a technology-shock generating function which depends on the rate of investment and has an S-shape form. This property of the technology-shock generating function may result in a multi-equilibrium situation. In contrast to the initial Arrow model, it is proved that the technological parameter can converge to a positive value, if the rate of investment is to tend to a sustainable level in the long-run. The economy, therefore can have a perpetual long-run growth rate without any exogenous growth in the labour force or in technology. The effects of savings on long-run growth are also analysed. It is shown that, for a specific form of the technology-shock generating function, multiple balanced growth paths for the economy may exist. Therefore, different saving regimes may lead to different long-run growth paths.

Note that this chapter concentrates on properties of the model, given a) a constant growth rate of capital , or b) a constant savings to output ratio. The following chapter will endogenize the savings to output ratio.

3.1 Introduction.

The importance of technological progress in explaining the long-run growth pattern of an economy is indisputable. In the traditional Solow model, it is exogenous technological change that serves as one important factor which can explain why an economy can grow at a sustainable rate in the long-run, despite diminishing marginal productivity of factors. The problem that remains is why this technological progress takes place, is technological change entirely exogenous or can it be affected by the economic activity, and if it does, then how?

The idea that the technological change is not totally exogenous is not new . The difference lies in the choice of variables and formalisation. Some unorthodox growth theorists in the 50s and 60s mentioned the possible relationship between technological change and the state of the economy. Kaldor (1957), for example, argued that:

> A society where technical change and adaptation proceed slowly, where producers are reluctant to abandon traditional methods and adopt new techniques, is necessarily one where the rate of capital accumulation is small. The converse of this proposition is also true; the rate at which a society can absorb and exploit new techniques is limited by its ability to accumulate capital.
>
> (Kaldor, 1957:595)

Arrow (1962) advanced the hypothesis that technological change can be associated with experience. He used *cumulative gross investment* as an index of experience, and argued that the change in stock of knowledge will affect productivity. That is because ' every new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli'. (Arrow, 1962:157). The state of technology, therefore, can be considered as a function of total capital stock. Sheshinski (1967) clarified this idea of Arrow and proposed an aggregate production function of the form for an individual firm: $y = F(k, A(K)l)$, where y is output, k is the firm's individual capital stock, and K is the total, or aggregate capital stock. Therefore, there exists a positive externality for a firm due to an increase in the capital stock of the other firms. The aggregate production function exhibits increasing returns. The technological change now becomes endogenous but the growth rate remains exogenous in the long-term.

Scott emphasises the role of investment as the cause of the advance of knowledge. To him,

> ' ... no ordinary investment consists *purely* of reduplication, and there is always present some element of novelty. Consequently, every investment forms part of the step-by-step process through which the advance of knowledge takes place'.
>
> Scott, 1993:34.

Furthermore, to Scott, ' both inventions of artefacts and developments of new forms of business organisation have resulted from preceding general investment' and not by some particular kind of investment.

Eltis (1963, 1993) analyses the relationship between the rate of technical progress and *the ratio of investment to output.* He developed a model, in which technical progress is endogenous to the investment process due to the stimulus of the latter to research and development activity and the Arrow learning effects which go along with continuous investment. The decomposition of the total effects of the investment process on technological change on research and development effect and learning effect is interesting and has a very important implication. Eltis argued that the world's technological leaders will obtain much of their technical advance from their own research and development , while backward economies which rely mainly on foreign technology will have a negligible research and development effect. However, there is no reason why the learning function should be stronger for the technical leaders (Eltis, 1993). The technical progress function, proposed by Eltis, is linear with respect to the ratio of investment to output. It follows, according to Eltis, that economies with different saving and investment propensities will have different rates of technical progress, and different steady growth rates. Therefore, saving and investment propensities will influence an economy's growth rate, even in the long-run, (Eltis, 1971). However, it can be shown that this proposition is valid only in very exceptional circumstances.

Suppose the production $f(k)$ is a Cobb-Douglas function and the technological progress function is linear, then $f(k)$ can be expressed as follows:

$f(k) = e^{bst}k^{\alpha}$

where k is the capital stock and s is the exogenous saving rate.

Investment now can be written as follows:

$$I = \dot{k} = sf(k) = se^{bst}k^{\alpha} \qquad \text{(1a)}$$

This represents a Bernoulli differential equation for k, so its solution can be found as follows:

$$k(t) = k_0 e^{\frac{bs}{1-\alpha}t}$$

It yields another expression for the investment

$$\dot{k} = \frac{bsk_0}{1-\alpha} e^{\frac{bs}{1-\alpha}t} \qquad \text{(1b)}$$

The difference between Equation (1a) and (1b) is the fact that the former expresses the *actual investment* for a given saving rate, while the latter represents the *desired investment* needed to keep the economy on a would-be balanced growth path. Given the initial capital stock $k(0) = k_0$, a saving rate which yields a balanced growth path must equate the actual and desired investment. In other words, this saving rate must satisfy the following condition:

$$sk_0^{\alpha} = \frac{sbk_0}{1-\alpha}$$

It is clear now that if $b \neq (1-\alpha)k_0^{\alpha-1}$, then the only balanced growth path is that with a zero rate. Therefore the proposition made by Eltis about the influence of saving and investment propensities on long-run growth is valid only in one very exceptional case when $b = (1-\alpha)k_0^{\alpha-1}$

The reason for this resides in the rigidity of assumptions about the constancy of the saving rate *and* the linearity in the relationship between technological change and investment. The model is static by nature, so it cannot provide any adjustment mechanism for the economy to move to an equilibrium. This problem is partly solved by King and Robson (1992) in a model which will be considered later.

The endogeneity of technological change has been intensively studied with the emergence of the new wave of endogenous growth theory. Romer (1986) in his first model, revised the Arrow learning-by-doing approach. The technological change in his model, is no longer a side-effect of the capital accumulation process but is a product of a new specialised sector producing new knowledge, intentionally. The increase in the stock of knowledge is a function of *the ratio of new investment in knowledge to total stock of knowledge*, and the overall stock of knowledge has positive external effects on the productivity of an individual firm and makes the aggregate production function of increasing returns. Under some additional assumptions on the production function and the knowledge creation function , the economy can grow at some positive, sustainable and even explosive rate . The advance of knowledge, in the Romer model, is characterised by the following differential equation:

$$\frac{\dot{k}}{k} = g(I / k)$$, where k is stock of knowledge, not physical capital.

Other factors, including physical capital, are assumed to be fixed. So the model, as Romer, himself realised, is the polar opposite of the usual model with endogenous accumulation of physical capital and no accumulation of knowledge. (Romer, 1986) . The model, therefore, can be interpreted as the special case of the two-variable models in which knowledge and capital are used in fixed proportions, and/or *k(t)* can be interpreted as a composite capital good, as in the Arrow model. This has led some (for example, Amable and Gullec (1993)) to believe that, in fact it is the physical capital that is implicitly used in the model. Furthermore, if the assumption about the fixed proportion between knowledge and physical capital is valid, then in the knowledge creation function one may use the rate of investment in *physical capital* instead of the rate of investment in knowledge, which is more difficult to observe and measure.

Finally, it is interesting to mention another model of endogenous technological change developed by King and Robson and named ' learning by watching'(1992, 1993). In this model, as in Eltis's model, *the ratio of new investment to output* is considered as a factor, which affects the change in technology. The rationale behind this choice of variables is the fact, as King and Robson argue, that 'there is a demonstration effect from observations of new ideas embodied in new investment projects to the level of output that can be produced from the existing stocks of capital and labour' (King and Robson, 1993). Furthermore, it is assumed that the probability distribution of the numbers of ideas observed by a representative agent is a Poisson distribution and the marginal benefit of observing an idea is decreasing in the number of observed ideas. The technological progress function which relates the productivity growth rate and the rate of investment, therefore, has a special S-shaped form: it is convex, when the investment rate is relatively small, and becomes concave when the rate of investment becomes larger. The model, due to this specific feature, leads to multiple equilibria for the steady growth rate. A low rate of investment generates little learning by watching, and hence a low rate of growth. Another equilibrium with a higher rate of investment leads to higher overall growth.

As noted previously, the King and Robson model is a natural extension of the Eltis model providing different microeconomic justification, which leads to an interesting non-linear technological progress function, and replacing the rigid assumption about the constancy of the saving rate by endogenizing it. Not surprisingly, the growth rate becomes endogenous, and the model can provide much richer implications.

3.2 Effect of technological change on TFP

In the following section, we will examine the implications of constructing a technological function and embodying it into a growth model.

First of all, we assume that technological progress is a function of the growth rate of a technology-enhancing factor. This factor can be investment in physical or human capital, imported capital goods for a developing country and so on. The reason why it is the growth rate rather then the stock of the technology-enhancing factor which results in a shift in total factor productivity as a measure of technological change has been considered partly by King and Robinson (1993). However, concerning the choice of variable for the technological progress function, our approach is closer to that of Romer; that is, we assume that the growth rate in productivity is affected by the intensity of a technology-enhancing factor measured by the ratio of new investment in this factor to its existing stock, not to the total output.

Furthermore, we assume that, while the change in the growth rate of a technology-enhancing factor can lead to an increase in TFP, this effect is falling over time. This reflects the reality that once a discovery is made , it has an immediate effect on production. However, as time passes, this discovery becomes out-dated and the benefit from using and implementing this invention declines. In other words, while new investment provides new stimulus for learning, this stimulus declines over time. In another interpretation, Shell (1967) argues that decay in technical knowledge is observed because of imperfect transmission of technical information from one generation of the labour force to the next. In this perspective, one may expect that, in order to have a sustainable increase in the TFP, the investment in technology-enhancing factors should be permanent.

Formally, we can model this process in the following simple model.

Consider a closed economy using capital and labour to produce a single good. For simplicity, we assume that the labour force is fixed. The production function is as follows:

$$F(t) = A(t).f(k(t))$$

where k is the capital stock

$A(t)$ is the TFP

The technological function A(t) is supposed to have both exogenous and endogenous components with the latter being a function of a technology-enhancing factor, that is

$$A(t) = A_0 e^{(\mu + z(t))t}$$

The most important component in this function is the function z(t) which is the subject of our consideration.

Let $\lambda(t)$ be the investment to capital ratio at time t, i.e $\lambda(t) = \dfrac{I(t)}{k(t)} = \dfrac{\dot{k}}{k}$

This investment to capital ratio can be considered as a measure of the intensity of investment in the technology-enhancing factor. The technological change induced by the new investment is supposed to be a function of $\lambda(t)$, say $g(\lambda)$, which reflects an increase in productivity due to the investment activity. However, this effect, as has been argued previously, decreases over time, and we assume that it is falling at a constant rate β. Without further investment, the output at time $t + \tau$, that is after τ period since the change in technology has occurred, under these assumptions can be expressed as follows:

$$F(t+\tau) = A(t+\tau)f(k(t+\tau)) = A_0 e^{(\mu+z(t+\tau))t} f(k(t+\tau))$$

The TFP at time $t+\tau$ is given by $A(t+\tau) = A_0 e^{(\mu+z(t+\tau))t}$ and $z(t + \tau)$ can be written as follows:

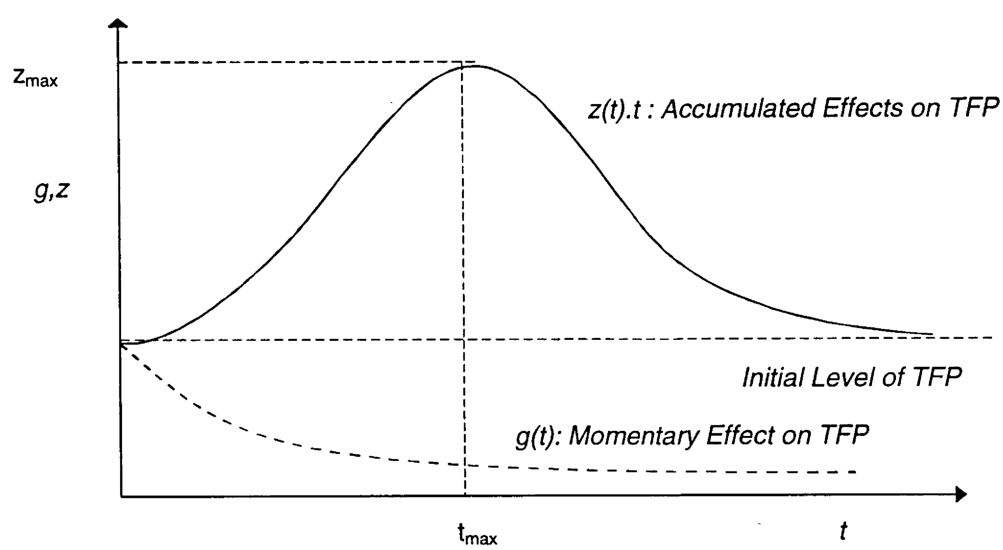
$$z(t+\tau) = e^{-\beta\tau}.\lambda(t)$$

It is not difficult to verify that the TFP attains its maximum value at the moment

$$\tau_{\max} = \frac{1}{\beta}$$

Therefore, the parameter β can be considered as the gestation period for the technological progress induced by an increase in the investment to capital ratio. The productivity continues to increase until it hits the maximum value at moment τ_{max}. After that the productivity begins to fall and approaches its initial, pre-shock level, (see Figure 3.1). This feature of the technology function makes it different from the approach adopted by King and Robinson (1993) where technological progress induced by investment has only a one-off, immediate effect on the total factor productivity. In this model, the change in technology, and hence in productivity, is affected by not only one particular shock, but by the whole history of the production and accumulation process. On the other hand, an increase in the investment ratio is supposed to have a permanent, although diminishing, effect on productivity as can be shown in the following diagram.

Figure 3.1- Effects of a single shock on productivity.



Consider now the more general situation, when the economy is affected by a series of productivity shocks. Suppose at moment t a single shock occurs with intensity λ_t. The momentary effect on the TFP, as has been argued previously, is measured by a function $g(\lambda)$, called the technology-shock generating function . The technological change induced by this series of shocks can be written as follows.

$$z(t) = \int_0^t g(\lambda_\tau) e^{-\beta(t-\tau)} d\tau = e^{-\beta t} \int_0^t g(\lambda_\tau) e^{\beta\tau} d\tau \quad (3.1)$$

The meaning of this formula is as follows. At moment t, the productivity is affected by the history of past shocks which have occured so far. The momentary effect of a shock which took place at moment τ is equal to $g(\lambda_\tau)e^{-\beta(t-\tau)}$, because it has been happening for $(t-\tau)$ periods of time. So the total change in productivity measured by function z(t) is simply the sum of all the changes induced by different shocks which took place at different times. It can be shown that the technological progress function z(t) defined in (3.1) is a solution to the following differential equation

$$\frac{dz}{dt} + \beta z(t) = g(\lambda_t), \text{ subject to } z(0) = z_0. \quad (3.2)$$

It should be noted that the usual assumptions on the nature of technological change used in the traditional Solow model fit this specification. For example, a stationary production function (without any change in TFP) is a special case, where $g(t) = 0$ for all t, and $z(0) = 0$. Suppose now that the economy is also affected by a single productivity shock with intensity equal to μ, that is $g(t)= 0$ for all $t> 0$ and $g(0)=\mu$. The momentary effect of this shock is the same for all periods of time. So the reduced technological progress function z(t) can be defined as follows:

$$z(t) = \lim_{\beta \to 0} z_\beta(t)$$

where $z_\beta(t)$ is a solution to the Equation (3.2), that is $z_\beta(t) = \mu e^{-\beta t}$. Therefore $z(t) = \lim_{\beta \to 0} z_\beta(t) = \mu$ for all t , which is required for the Solow neutral technological change.

The technological progress function specified above can be used in many different models depending on which factor is considered as technology-enhancing. In what follows, we will apply this specification for a simple case where the technological change is assumed to be affected by the corresponding capital accumulation process.

Investment in this case is considered as a technology-enhancing factor .

3.3 Model.

The dynamics of the economy now can be fully described by the following system of equations.

$$F(k) = A_0 e^{z(t).t} k^{\alpha} \quad (3.3)$$

$$\frac{dz}{dt} + \beta z(t) = g(\lambda_t) \quad (3.4)$$

$$\lambda_t = \frac{dk}{kdt} \quad (3.5)$$

where $z(t)$ is the technological progress function;

λ_t is the investment to capital ratio;

$0 < \alpha, \beta < 1$

Given some initial conditions regarding the capital stock, share of factors in total output and the underlying technological function $g(\lambda)$, the growth path of the economy is entirely determined by sequences of the capital accumulation path, $\{\lambda_t\}$, t= 0, 1, 2 ... It is assumed that the capital accumulation path is irreversible, that is $\lambda_t \geq 0$ for all t.

The level of consumption at moment t is defined by

$$c(t) = F_t(k_t) - \lambda_t k_t$$

For further exposition of the model, we adopt some definitions, most of which have been used by Sheikman, Brock (1976) and McKenzie (1986).

3.3.1 Some definitions

Definition 1. A capital accumulation path is called expansible if $\lambda_t > 0$ for all t > T.

Definition 2. A capital accumulation path is called feasible if $c_t \geq 0$ for all t.

Definition 3. A capital accumulation path is called perpetual if it is feasible and $\lim_{t \to \infty} \lambda_t = \lambda^* > 0$

Obviously, a perpetual accumulation path is necessarily an expansible and feasible one, but the opposite is not always true. An expansible path may not be a perpetual one. This can be shown in the case of the traditional Solow model with and without exogenous technological change. For example, in the case of the simplest model without technological change, the underlying production function is of the following form:

$$F(k) = A_0 k^{\alpha}$$

It is easy to show that, in this case, there is no perpetual accumulation path for the economy. In fact, let $\{\lambda_t\}$, t= 0, 1, 2 be an expansible and feasible path. Then

$$k(t) = k_0 e^{\int_0^t \lambda(\tau)d\tau} \quad \text{and} \quad F(t) = F_0 e^{\alpha \int_0^t \lambda(\tau)d\tau}$$

The consumption at time period *t, c(t)* is, therefore defined as :

$$c(t) = F(t) - \dot{k}(t) = F_0 e^{\alpha \int_0^t \lambda(\tau)d\tau} - \lambda(t) k_0 e^{\int_0^t \lambda(\tau)d\tau} =$$
$$e^{\alpha\mu(t)}\left[F_0 - \lambda(t) k_0 e^{(1-\alpha)\mu(t)}\right]$$

where $\mu(t) = \int_0^t \lambda(\tau)d\tau$

Suppose that $\lim_{t\to\infty} \lambda_t = \lambda^* > 0$, then there exist a time period T and a positive value ε, for which $\lambda(t) > \varepsilon$ for all t > T. It follows that :

$$c(t) < A(F_0 - Be^{(1-\alpha)\varepsilon t})$$

where A and B are some well-defined positive values.

As t increases c(t) will be negative. The accumulation path is not feasible. That means all possible expansible paths have to tend to the zero level: the economy will cease to grow in the long-run.

However, in the case where the rate of investment has a positive effect on productivity, it is possible that under some conditions, the economy can be kept on the growth track forever. It is a necessary condition for an endogenous growth path as we will show in what follows

3.3.2 Possibility of a perpetual growth path

The mechanism which underlies the growth pattern in the model specified above can be explained as follows. As the economy grows (more resources are being used for capital accumulation at the expense of consumption), the marginal productivity of capital would fall due to diminishing returns to factors. On the other hand, the investment activity also has a positive effect on the total productivity, because of the novelty in the production process which is accompanied by this investment process. This counter-effect will lessen the speed of the fall in the marginal productivity of capital. The final outcome on the overall productivity depends on the relative strength of these two opposite forces. If the fall in productivity due to the first effect cannot be fully compensated by the rise in the productivity due to the induced technological change, then the economy will end up with no growth in the long-run. Conversely, if the induced technological change can be maintained at some significant level in the long-run, it will make plausible the possibility of having a perpetual growth path. The following lemma will provide a condition for such a possibility:

Lemma 1: *If the growth rate of capital stocks converges to a positive level, then the induced technological parameter also converges to a positive value.*

Proof:

In fact, by definition of the technological parameter, we have

$$z(t) = e^{-\beta t} \int_0^t g(\lambda_\tau) e^{\beta \tau} d\tau$$

Applying the L'Hopitale rule to z(t) leads to

$$\lim_{t\to\infty} z(t) = \lim_{t\to\infty} \frac{\frac{d}{dt}\int_0^t g(\lambda_\tau)e^{\beta\tau}d\tau}{\frac{d}{dt}e^{\beta t}} = \lim_{t\to\infty} \frac{g(\lambda_t)e^{\beta t}}{\beta e^{\beta t}}$$

Given the fact that $\lim_{t\to\infty} \lambda_t = \lambda^* > 0$, it implies $\lim_{t\to\infty} z(t) = \frac{g(\lambda^*)}{\beta} > 0$ ■ *QED*

This lemma has two important implications: Firstly, it confirms the importance of the 'learning effects' of the accumulation process, that is the effects of a sustainable growth in investment on productivity. If growth in investment can be maintained at a positive level in long-run, it can induce a sustainable increase in the total productivity which is equivalent to exogenous technological change in the traditional Solow model. However, unlike the Solow model, the magnitude of this change in productivity is endogenous, rather than exogenous, because it depends on the investment ratio. The larger is the rate of investment, the larger is the induced rise in the productivity.

The second implication concerns the pattern of the accumulation process and the long-run trend of growth. According to Lemma 1, the long-run trend of growth in productivity depends on the long-run trend of the accumulation process. From this perspective, two accumulation processes $\{\lambda_1(t)\}$ and $\{\lambda_2(t)\}$ are equivalent if they converge to the same level, that is:

$$\lim_{t\to\infty} \lambda_1(t) = \lim_{t\to\infty} \lambda_2(t) = \lambda.$$

As long as our main concern is the long-run growth pattern of the economy, this implication allows us to make some appropriate assumptions, where needed, on some initial segments of the accumulation process, without changing the main qualitative results of the growth process. In particular, it is useful to start with an accumulation process with constant growth rate of investment, that is to consider the long-run properties of the process, where the accumulation path is given by the sequence $\{\lambda(t) = \lambda\}$ for all t.

3.3.3 The existence of a balanced growth path

The production function, described in Equation (3.3), is non-stationary. That makes it difficult to study the dynamics of the whole system (3.3)-(3.5). So it is useful to transform the variables measuring the output and capital into levels per efficiency unit of labour. The new variables can now be defined as follows:

$$w(t) = \frac{z(t)}{1-\alpha}$$

$$G(t) = \frac{F(t)}{e^{w(t).t}}$$

$$h(t) = \frac{k(t)}{e^{w(t).t}}$$

where *w(t)* can be considered as a modified technological progress function

G(t) is the output , measured in terms of the efficiency unit of labour

h(t) is the capital stock, measured in terms of the efficiency unit of labour

The dynamic behaviour of the economy, can now be described by the following system of equations:

$$G(h) = h^{\alpha} \qquad (3.6)$$

$$\frac{dw}{dt} = \frac{g(\lambda_t)}{1-\alpha} - \beta w \qquad (3.7)$$

$$\frac{\dot{h}}{h} = \lambda_t - (w + \dot{w}t) \qquad (3.8)$$

$$\lambda_t = \frac{dk}{kdt} \qquad (3.9)$$

The modified production function now becomes stationary, but the system is still non-autonomous. However, the steady-state (or stationary) values of the modified technological parameter w^* , that of the transformed capital stock h^* and the investment ratio λ^* can be found by solving Equations (3.7)-(3.9). This yields the following result:

$$\lambda^* = w^*$$

and λ^* is a solution to the following equation:

$$\frac{g(\lambda)}{1-\alpha} = \beta\lambda \qquad (3.10)$$

It should be noted that, under the assumptions on the technology-shock generating function $g(\lambda)$ made in the previous section, Equation (3.10) has, at least, one solution $\lambda=0$. If it is the only solution to Equation (3.10), the model is not interesting, because, despite the assumption on possible endogenous growth, the only feasible balanced growth path has a zero growth rate in the long-run, and there is no difference between this kind of model and the traditional one. Therefore, for the model to yield a positive solution, additional assumptions on the function $g(\lambda)$ are needed. In what follows, we make two important assumptions:

Assumption G1:

$$\left.\frac{dg}{d\lambda}\right|_{\lambda=0} > (1-\alpha)\beta$$

Assumption G2: The function $g(\lambda)$ has an S-shaped form.

A technological progress function which satisfies Assumption G2 was proposed by King and Robson (1993). An important property of this kind of function is that it has first increasing and then decreasing returns to the independent variable and, hence, reflects the non-linearity of the relationship between productivity growth and the rate of investment. Two possible justifications for this assumption are shown in the next section. The first is proposed by King and Robson and based on the learning by watching or the demonstration effects of investment. The second one is based on the vintage model and the effects of technology diffusion on production.

'Learning by watching' effects: King and Robson hypothesis

King and Robson (1993) proposed one explanation for an S-shaped technological progress function. Although some specifications are different to this study, especially as to what is chosen to be a measure of the technology-enhancing activity, some arguments are useful and can be applied.

The economy under consideration consists of a large number of individuals who are divided into two classes: 'teachers' who have ideas and 'learners',who learn by watching what other people do. Typically, the teachers are carrying out some investment projects and the learners are 'watching', so the relative proportions of the two types are X and 1-X, where X is the investment rate that is the ratio of investment to the total output. By investing, the teachers may uncover new ideas and the probability of the number of ideas observed by a representative agent is a Poisson distribution with mean $\mu = \frac{\lambda X}{1-X}$, where λ is the average number of new ideas discovered by one investment project. Therefore the probability of contact with N ideas is given by

$$prob(N) = \frac{\mu^N e^{-\mu}}{N!}$$

On the other hand, by observing a new idea, the learners can increase their productivity. This benefit from watching, measured in terms of productivity growth gains, is positively relative to the number of ideas encountered, N, but its marginal benefit is decreasing in N. King and Robson consider a very simple case when this total benefit $b(N)$ has the form:

$b(N) = a(1-e^{-bN})$ where a and b are positive constants.

Therefore, the induced technical progress function which represents the unconditional (expected value of) benefit from learning by watching as a function of the investment rate X is expressed as follows:

$$\phi(X) = \sum_{N=0}^{\infty} b(N) prob(N) = \sum_{N=0}^{\infty} a(1-e^{-bN}) \frac{\mu^N e^{-\mu}}{N!}$$

Finally, the technical progress function may be written as follows:

$$\phi(X) = a\left\{1 - \exp\left(\frac{\lambda X(e^{-b}-1)}{1-X}\right)\right\}$$

It is not difficult to show that this particular function has an inflexion point at $\overline{X} = 1 - \frac{\lambda(1-e^{-b})}{2}$. Therefore, if $\lambda < 2$ then the technical progress function has an S-shaped form. However if the average number of ideas embodied in an investment project is quite large, the technical progress function would be strictly concave over the positive quadrant. This represents one of the drawbacks of this specification.

In the following we suggest another explanation to why the technological function may have an S-shaped form. This specification is based on a modified vintage model and the analysis of effects of technology diffusion on productivity. Because this analysis is at the aggregate level we shall not provide detailed consideration of such phenomena as intra- or inter-firm diffusion of technology. It should also be noted that, in spite of the similarity in the functional form between the technology-shock generating function here and the diffusion function, we are in different analytical positions: the former deals with the possible effects of implementing new technology or learning new ideas *at a particular time*, while the latter deals with the speed of spreading new technology (and its effects) *over time*.

A modified vintage model: Diffusion effects

To see how the assumption about the S-shape of the technology-shock function may be well justified, let's consider the following specification.

As is widely accepted, investment always embodies some novelty and new ideas. This may be in the form of new capital goods or new products . It may involve some kind of innovation or imitation. In any case, one unit of the newly invested physical capital good is supposed to be more efficient than the older one, in the sense that it produces more output. Therefore, the capital stock can be viewed in two ways: in its physical form as we observe it, or in the effective form which takes into account the effects of the embodied technological change. Total output, therefore, can also be considered in two ways. It can be considered as a function of the physical capital stock or alternatively, as a function of the effective one. In other words, the production function can be expressed in the following two forms

$F = Af(K_e)$ or $F = A(t)f(K)$

where A is the total factor productivity

K_e is the effective capital stock

K is the physical capital stock

In the function of the effective capital stock, the TFP term is assumed to be given because all possible changes in technology are supposed to be captured in the effective capital. In the second expression the total factor productivity does depend on the investment activity.

Suppose further that the effective capital stock is measured in terms of the physical capital stock at the beginning. i.e at $t = t(o)$

The extent of technology diffusion (intra-firm or inter-firm) usually is measured by the ratio of output which is produced by the new technology to the total. Alternatively, as Stoneman suggested one may use the proportion of a firm's capital stock that is of new type machines as the measure of technology diffusion, (Stoneman ,1995). At the aggregate level, the ratio of new investment to the total capital stock can be used as a proxy of technology diffusion, although not all investment is devoted to acquiring new machines. On these grounds, one may assume that the effectiveness of the new capital depends on the extent of technology diffusion: the wider is the new technology adopted, the more efficient becomes the new investment because not only does the new technology have a direct effect on productivity but it also brings about some other positive externalities: creating new incentives and a favourable environment for learning and improving skills...The effective capital stock, therefore can be formally determined as follows:

$$K_e(1) = (1 + a(d))I + K(0) = K(1) + a(d)I = K(1)(1 + d * a(d))$$

where *K(1)* is the physical capital stock at time period 1

$K_e(1)$ is the effective capital stock at time period 1

K(0) is the physical capital stock at time period 0

I is the amount of investment

$\lambda = \dfrac{I}{K(0)}$ is the rate of growth of capital stock.

$d = \dfrac{I}{K(1)}$ is a measure of the technology diffusion.

a(d) is a measure of the effectiveness of new capital stock due to technology diffusion

The function *a(d)* is assumed to have a positive first derivative and a negative second derivative. That is the effectiveness of new capital is positively related to the extent of diffusion but with diminishing speed.

The output at time 1, therefore, is as follows:

$$F(1) = A_0 K_e^{\alpha}(1) = A_0[1 + da(d)]^{\alpha} K^{\alpha}(1)$$

It implies that the total factor productivity in the next period, as we would observe, will increase due to the investment activity and is given by

$$A_1 = A_0[1 + d * a(d)]^{\alpha}$$

Note that the growth rate of the capital stock and the extent of the technology diffusion are related by the following

$$d = \frac{\lambda}{1+\lambda}$$

The rate of change in the total factor productivity, hence, is a function of the rate of growth in the capital stock. The corresponding technology-shock function defined so far, has the following form:

$$g(\lambda) = \left[1 + \frac{\lambda}{1+\lambda} * a(\frac{\lambda}{1+\lambda})\right]^{\alpha} - 1$$

The function g(.), therefore, may exhibit the S-shape as can be shown in the following simple case.

The following simple example shows the possibility of the existence of an S-shaped technological progress function . The effectiveness of the investment function has the form : $a(\ d\ ) = d^{\beta}$. In other words, the elasticity of the effectiveness with respect to the growth rate of capital is constant. The technology-shock generating function g(.), therefore is given by:

$$g(\lambda) = \left(1 + \frac{\lambda^{1+\beta}}{(1+\lambda)^{1+\beta}}\right)^{\alpha}$$

where α is the share of capital in the total output

β is the elasticity of the effectiveness with respect to the growth rate of capital.

Table 3.1 Inflexion points for different hypothetical technological functions

$$g(\lambda)=\left(1+\frac{\lambda^{1+\beta}}{(1+\lambda)^{1+\beta}}\right)^{\alpha}-1 \text{ (values of } \lambda \text{ in percentages).}$$

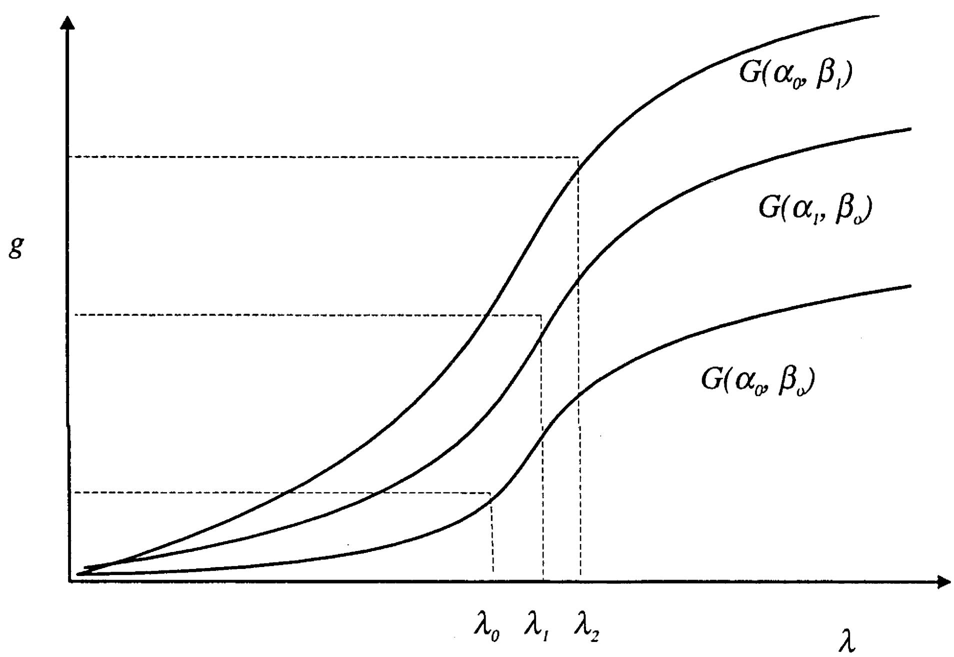
	α=0.35	α=0.45	α=0.6
β=0.3	12.7	12.9	13.5
β=0.4	17.1	17.5	18.1
β=0.5	21.6	22.1	22.7
β=0.6	26.1	26.7	27.5
β=0.7	30.7	31.3	32.2
β=0.8	35.5	35.9	36.8

For the function *g(.)* to be S-shaped, it is necessary for positive value of λ to exist such that the second derivative equals zero at this point λ, and this second derivative changes its sign from positive to negative while passing through this inflexion point. For the above specified function, this property in principle, can be proved analytically, but it is quite complicated because, the corresponding equation for finding the appropriate inflexion point may not have an analytical solution. For our exposition purpose here, it is possible (and much easier) to do this numerically.

Table 3.1 shows the values of the inflexion points solved numerically for each combination of α and β. For example, if the share of capital in the total output is 0.35 and the elasticity of the effectiveness of investment with respect to the growth rate of capital is 0.3 then the inflexion point is at λ = 12.7%

The position of inflexion points depends on the values of α and β. Figure 3.2 shows this dependency. For an initial combination (α_0, β_0) the technology-shock generating function is represented by the curve G(α_0, β_0) and the inflexion point is λ_0. An increase in either α or β will put the curve up to G(α_1, β_0) if α is to change or to G(α_0, β_1) if β is to change. In both cases, the inflextion point will move to the right.

Figure 3.2 Different hypothetical technological functions with different combinations of α and β



Assumption G1 can ensure a positive solution to Equation (3.10), while Assumption G2, as can be seen in Figure 3.3 may lead to a multi-equilibrium situation. This situation will have some quite interesting implications and will be considered in the next sections.

Suppose, for simplicity, that the capital accumulation process is given by a simple sequence {$\lambda(t) = \lambda$}for all t. The modified technological progress function w(t) can be found explicitly, by solving the corresponding differential equation in (3.7). It yields:

$$w(t)=\frac{g(\lambda)}{(1-\alpha)\beta}(1-e^{-\beta t}) \quad (3.11)$$

and $\dot{w}(t)=\dfrac{g(\lambda)e^{-\beta t}}{(1-\alpha)}$ for all t>0

The evolution of the capital stock, measured in terms of the efficiency unit of labour *h(t)* is characterised by the following differential equation:

$$\frac{\dot{h}}{h}=\lambda_t-(w+\dot{w}t)=\left[\lambda-\frac{g(\lambda)}{(1-\alpha)\beta}\right]+\frac{g(\lambda)e^{-\beta t}}{(1-\alpha)\beta}(1-\beta t) \quad (3.12)$$

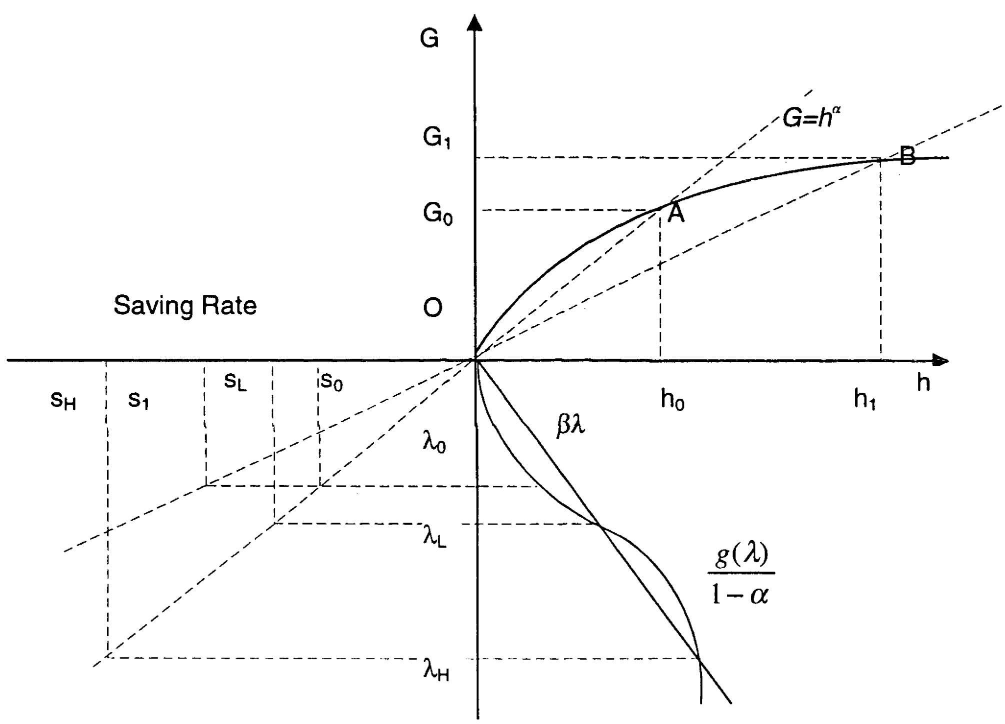
The second term in this expression will tend to zero as t approaches infinity. Therefore, for a large enough *t*, the following holds:

$$\begin{cases}\dot{h}>0 \ \text{ if } \ \lambda(1-\alpha)\beta>g(\lambda)\\ \dot{h}<0 \ \text{ if } \ \lambda(1-\alpha)\beta<g(\lambda)\end{cases} \quad (3.13)$$

The feasibility of the capital accumulation path with a constant rate of investment given by λ, can be analysed by using the diagram in Figure 3.3.

In this diagram, the relationship between output and capital stock (both are measured in terms of the efficiency unit of labour) is displayed in the first quadrant, that between saving rate and investment rate, in the third, and finally, that between the rate of investment and technological progress in the fourth. Because, the rate of investment to capital is assumed to be exogenously given, the corresponding saving rate, in this diagram, should be understood as *a required saving rate* , needed to support this accumulation process. At any given time period, this required saving rate depends on the rate of investment and the output-to-capital ratio. The larger is the rate of investment, and/or the lower is the output-to-capital ratio, the larger will be this required saving rate. By Definition 3, a capital accumulation path is feasible if the required saving rate is alway between 0 and 1. Otherwise, if starting from some time period T, this required saving rate is greater than 1, then the accumulation path is not feasible.

Figure 3.3. The rate of investment and growth.



Suppose the economy is initially at point A, with initial capital stock h_0, and initial output equal to G_0. Suppose further that the economy sets up an objective to keep the investment ratio constant over time at a constant rate λ_0;. The required saving ratio is s_0, (given by the value at the intersection of a line passing through the initial point A and the origin, and the line $\lambda = \lambda_0$ in the third quadrant). In this example, we assume that the technology-shock generating function is S-shaped, so there exist two equilibrium points corresponding to two balanced growth path with the rates of investment being equal to λ_L and λ_H. The required saving rates are s_L and s_H, respectively. The steady-state values of capital for these two cases are the same and equal to h_0.

The feasibility of other than equilibrium rates, can be considered in two separate cases:

Case 1: $0 < \lambda < \lambda_L$ or $\lambda > \lambda_H$. In this case , the rate of investment is bigger than the change in productivity induced by the investment activity, because

$$\lambda > \frac{g(\lambda)}{(1-\alpha)\beta}$$

The dynamics of the effective capital stock is described by Equation (3.13). As noted before, except for some initial periods, this capital stock tends to increase over time. So, after some periods, the economy will move from A to a new point B with higher capital stock and higher output. However, due to the diminishing marginal productivity of the underlying production function, the output to capital ratio tends to decline. The rise in productivity, due to the investment activity can slow down this process, but cannot reverse it. Therefore, in order to keep the accumulation process on track, the required saving rate would have to increase over time. But it would be impossible, because the required saving rate will eventually exceed one as the economy evolves. The supposed accumulation path, therefore, is not feasible.

Case 2: $\lambda_L. < \lambda < \lambda_H$. In this case , the rate of investment is less than the change in productivity induced by the investment activity, namely,

$$0 < \lambda < \frac{g(\lambda)}{(1-\alpha)\beta}$$

The dynamics of the economy is similar to that in Case 1, but the efficient capital stock *h(t)* will move in the opposite direction and eventually tends to zero. The actual capital stock *k(t)* will increase overtime at the constant rate λ. The long-run production function exhibits an increasing return to capital, because the output grows faster than the capital stock, due to a significant rise in productivity. The accumulation path is feasible because the required saving rate is always in the range between zero and one. Furthermore, this saving rate will approach zero, explaining the fact that the fraction of the total output needed to keep the economy on track with a constant rate of investment becomes smaller and smaller because the total productivity increases at a rate higher than that of capital stock.

The results of the above non-formal analysis can be summarised in the following Proposition 3.1.

Proposition 3.1: *If an accumulation path of an economy, described by the system of Equations (3.6)-(3.9) has a constant rate of growth λ, then:*

a) The path represents a balanced growth path for the economy, if λ is a non-zero solution to the equation: $\lambda = \dfrac{g(\lambda)}{(1-\alpha)\beta}$

b) The path is infeasible if $\lambda > \dfrac{g(\lambda)}{(1-\alpha)\beta}$

c) The path is feasible if $\lambda < \dfrac{g(\lambda)}{(1-\alpha)\beta}$

It should be noted that these results can be proved formally. In what follows, we will give a formal proof for Case b . The rest is straightforward.

Suppose an accumulation path with a constant rate of growth λ, satisfying the condition:

$$\lambda > \frac{g(\lambda)}{(1-\alpha)\beta}$$

Along this path, the output $F(t)$and capital stock $k(t)$ are given by:

$$k(t) = k_0 e^{\lambda t}$$

$$F(t) = A_0 e^{z_t t} k_t^{\alpha} = F_0 e^{\left[\frac{g(\lambda)}{\beta}(1-e^{-\beta t})+\alpha\lambda\right]t}$$

The required saving rate needed to keep the economy on this hypothetical accumulation path, therefore, is given by

$$s(t) = \frac{\lambda k_t}{F(t)} = \frac{\lambda k_0}{F_0} e^{\left[(1-\alpha)\lambda - \frac{g(\lambda)}{\beta}(1-e^{-\beta t})\right]t}$$

By assumption $\lambda > \dfrac{g(\lambda)}{(1-\alpha)\beta}$. This implies that $\lim_{t\to\infty} s(t) = \infty$. The accumulation path is, therefore, infeasible. ■ *QED*

3.3.4 Saving and growth.

In the previous section, we considered the feasibility of an accumulation path with a constant rate of growth in investment, given exogenously. We have proved that if technological change is affected by the rate of investment, then the economy can grow at some sustainable rate in the long-run if the change in productivity induced by the investment activity is not smaller than the rate of investment.

However, the assumption about the constancy of the rate of investment is very arbitrary. It would be more reasonable to make the assumption about the constancy of the saving ratio and to consider the possible relationship between the growth rate and this saving rate as it was in the early version of the Solow model. This analysis will be carried out in this section.

Once again, consider an economy, described by the system of Equation (3.6)-(3.9). Assume that the saving rate is exogenously given and constant over time. Suppose this saving rate is equal to s. In this case, the investment to capital ratio is given by :

$$\lambda_t = \frac{dk}{kdt} = \frac{sF}{k} = \frac{sG}{h} = sh^{\alpha-1}$$

The system of Equations (3.7)-(3.8) can be rewritten as follows:

$$\dot{w} = \frac{g\left(sh^{\alpha-1}\right)}{1-\alpha} - \beta w \quad (3.14)$$

$$\frac{\dot{h}}{h} = sh^{a-1} - (w + \dot{w}t) \qquad (3.15)$$

The dynamics of the whole system is entirely defined by this system of differential equations. One of the difficulties in dealing analytically with this system is the fact that it is a non-autonomous one. However, some important properties of the long-run trend in the dynamics of the system can be analysed by graphical and simulation techniques, which will be undertaken in this section. Furthermore, the steady-state of the system can be solved analytically as follows:

Let (w^*, h^*) be steady values of the system (3.14)- (3.15), then (w^*, h^*) is nothing but a solution to the following system of equations:

$$\begin{cases} g(sh^{\alpha-1}) = (1-\alpha)\beta w \\ sh^{\alpha-1} = w \end{cases} \qquad (3.16)$$

The first condition means that the technological parameter $w(t)$ is in its steady state if a loss in knowledge during its transmission process from one generation of workers to another is fully offset by newly acquired knowledge due to the investment activity. The second condition implies that the efficient capital stock will reach its steady value if the growth rate of physical capital stock is totally matched with the rate of change in productivity.

The steady-state value of the technological parameter, w^*, can be found by solving the following equation:

$$\frac{g(w)}{(1-\alpha)\beta} = w \qquad (3.17)$$

The corresponding steady value for the efficient capital stock h^*, can now be determined as follows:

$$h^* = \left(\frac{w^*}{s}\right)^{1/(\alpha-1)}$$

3.4 Effects of saving on growth.

Once again, it can be seen that for a specific form of the technology-shock generating function (for instance, the S-shaped one), there may exist multiple (two) solutions to Equation (3.17). This yields two equilibria with different long-run growth rates. The lower growth path, then, would require a lower saving rate s_L, while the higher growth path certainly requires a higher saving rate s_H. If initially, the saving ratio is equal to one of these values, the economy will embark immediately on one of these two balanced growth paths, along which both output and capital will grow at the same rate. The long -run output-capital curve is linear, with the slope being bigger for the higher growth path, and smaller for the lower one.

It is more interesting to address the following two questions:

1- If the pre-set saving rate differs from these two balanced values, how will the economy evolve? To which balanced growth path, will the economy move?

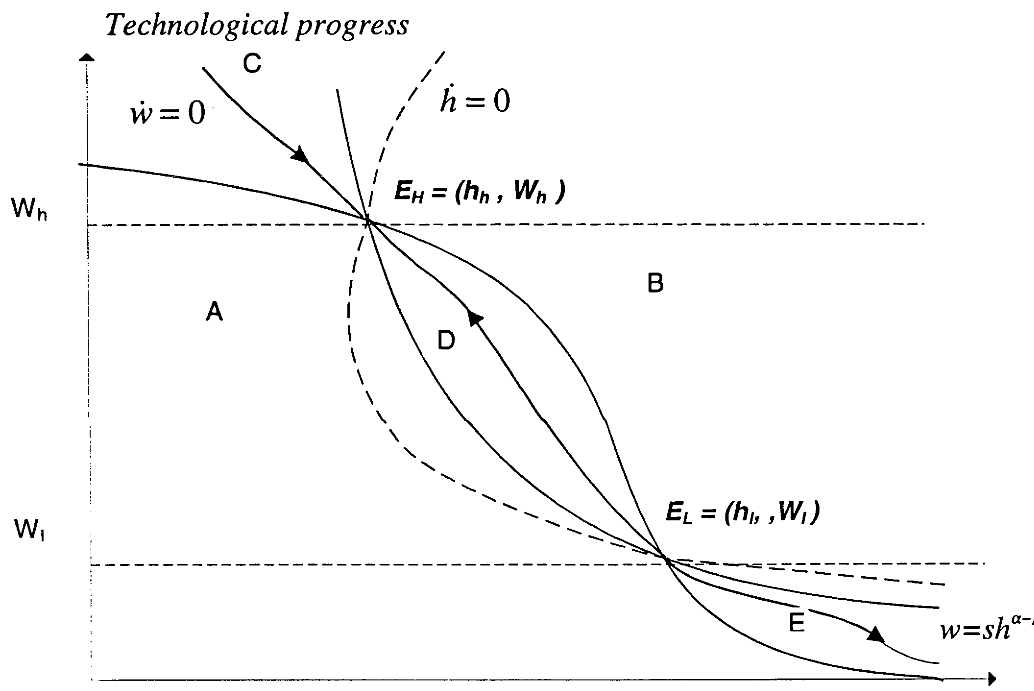
2- Suppose, the economy is already on a balanced growth path and there is a one-off change in the saving rate. Does it affect the long-run growth rate of the economy? What does the transitional adjustment look like?

In the traditional Solow model when technology is totally exogenous, the answer to the second question is negative: any change in the saving rate has only a temporary effect on the growth rate but not on the long-run one. However, in the model incorporating the technological progress function described above, it is possible that the change in saving rate may lead the economy to different steady-states characterised by different long-run growth rates. This proposition will be demonstrated by using a graphical analysis and a simulation experiment.

A phase diagram analysis :

Recall that the dynamics of the economy in this model is entirely determined by the system of differential Equations (3.14)-(3.15). This is a non-autonomous system because the right-hand side of Equation (3.15) contains both time-invariant and time-dependent components. It makes a phase diagram analysis more complicated, since the locus $\dot{h} = 0$ in the *(h, w)* space shifts over time as the system is evolving towards a steady-state. However, given the relative simplicity of the time-dependent component in Equation (3.15), the position of this locus is very closely related to that in the underlying autonomous system. Therefore, the direction of movement of the locus over time is fairly predictable.

Figure 3.4: Savings and growth. A phase diagram analysis



In Figure 3.4, the locus $\dot{h} = 0$ is represented by the *HH'* curve, while the locus $\dot{w} = 0$ is represented by the *WW'* curve. Under our assumption about the S-shaped technology-shock generating function g (λ), these two loci intersect at two points E_L and E_H. Since Equation (3.15) contains a time-dependent component, the locus $\dot{h} = 0$ will shift over time. However, this locus will always pass through the two above mentioned equilibrium points. The whole phase space is divided into five distinct regions denoted by A, B, C, D, E. Formally, these five regions can be defined as follows:

$$
\begin{aligned}
A &= \{(h,w): \dot{h} \geq 0, \dot{w} \geq 0\} \\
B &= \{(h,w): \dot{h} \leq 0, \dot{w} \leq 0\} \\
C &= \{(h,w): w \geq W_h, \dot{h} \geq 0, \dot{w} \leq 0\} \\
D &= \{(h,w): W_h \geq w \geq W_l, \dot{h} \leq 0, \dot{w} \geq 0\} \\
E &= \{(h,w): w \geq W_l, \dot{h} \geq 0, \dot{w} \leq 0\}
\end{aligned}
$$

It is not difficult to show that

$$
\begin{aligned}
A &\subseteq \overline{A} = \{(h,w): sh^{\alpha-1} \geq w, g(sh^{\alpha-1}) \geq (1-\alpha)\beta w\} \\
B &\subseteq \overline{B} = \{(h,w): sh^{\alpha-1} \leq w, g(sh^{\alpha-1}) \leq (1-\alpha)\beta w\} \\
C &\supseteq \overline{C} = \{(h,w): w \geq W_h, sh^{\alpha-1} \geq w, g(sh^{\alpha-1}) \leq (1-\alpha)\beta w\}
\end{aligned}
$$

$$
\begin{aligned}
D &\subseteq \overline{D} = \{(h,w): W_h \geq w \geq W_l, sh^{\alpha-1} \leq w, g(sh^{\alpha-1}) \geq (1-\alpha)\beta w\} \\
E &\supseteq \overline{E} = \{(h,w): w \geq W_l, sh^{\alpha-1} \geq w, g(sh^{\alpha-1}) \leq (1-\alpha)\beta w\}
\end{aligned}
$$

It can be seen that the position of five sets $\overline{A}$, $\overline{\mathrm{B}}$, $\overline{\mathrm{C}}$, $\overline{\mathrm{D}}$, and $\overline{E}$ is time-invariant and can serve as a benchmark for sets A, B, C, D and E. Namely, the sets $\overline{A}$, $\overline{\mathrm{B}}$, and $\overline{\mathrm{D}}$ are upper bounds for A, B and D, respectively, while $\overline{\mathrm{C}}$, and $\overline{\mathrm{E}}$ are lower bounds for C and E. Furthermore, this implies that the movement of the non-autonomous system, while being in different regions is not much different from the dynamics of the underlying autonomous system. The only difference is that the time-dependent component in Equation (3.15) will *slow down* the movement of the system in regions A, B and D, and *speed it up* if the system is in regions C and E.

Suppose the economy now is in region A with the initial effective capital stock h_0 and technological parameter w_0. The rate of investment is given by $sh_0^{\alpha-1}$. Because, the newly acquired knowledge is larger than the loss associated with knowledge transmission, productivity will increase at a rate higher than the rate of investment. Therefore, both the effective capital stock and technological parameter increase; the economy moves upwards. As time passes, if the economy enters region D, then it will move to the higher growth path E_H. It may intercept region E as well. In this case, the economy will eventually move to the no-growth state.

It is clear now that, in this model, the higher growth path is stable, while the lower one is unstable. Therefore, the starting point of the system is very relevant in determining in which equilibrium the economy will end up. The two above specified problems, can now be considered in this context.

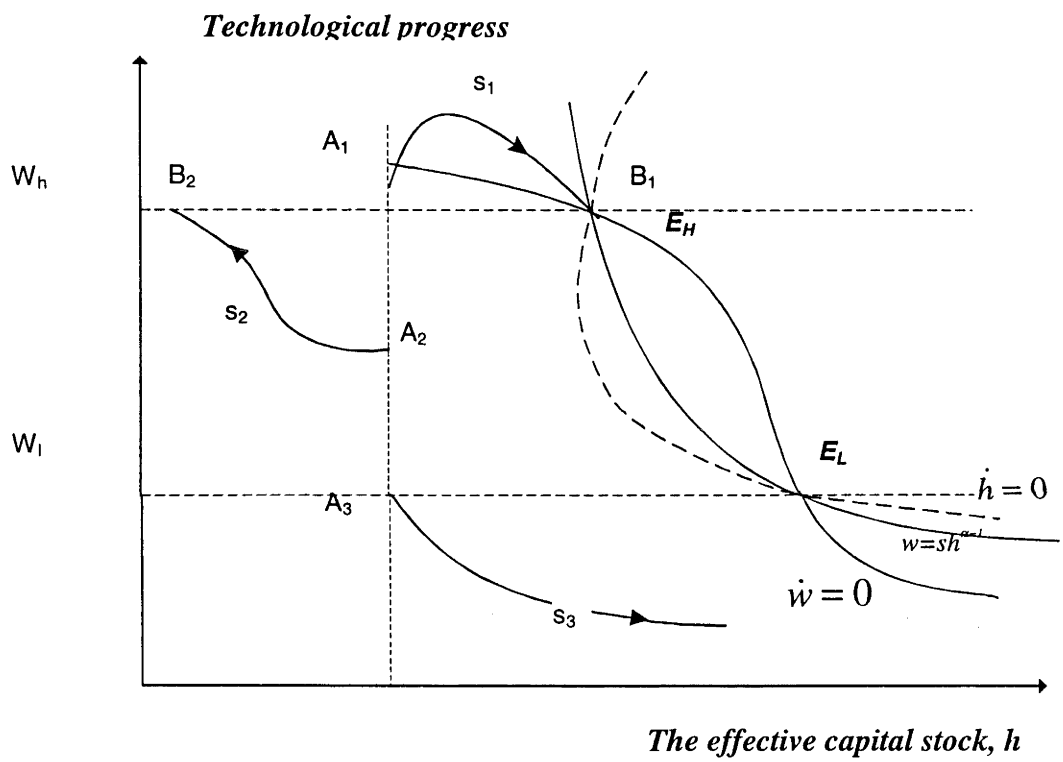
3.4.1 Effects of saving on growth: A graphical analysis

Firstly, we consider a hypothetical situation, when an economy starts to evolve from a moment $t_0 = 0$, with initial effective capital stock $h(0) = h_0$. Prior to this moment, technology is entirely exogenous, or at least is not affected by the investment activity. The saving ratio is exogenously given and equal to s. The rate of investment $\lambda(t)$, therefore, is a function of this saving rate and the stock of efficient capital and is given by

$$\lambda(t) = sh^{\alpha-1}$$

As discussed previously, given a specific form of the technology-shock generating function $g(\lambda)$, there exist two saving rates which enable the economy to settle on two distinct balanced growth paths immediately. The higher growth path, characterised by point E_H in Figure 3.5 is ensured by the saving rate s_H, while a saving rate equal to s_L leads to a lower balanced growth path, characterised by the steady-state at point E_L. For any other exogenously given saving rate, one and only one of the following three cases will hold:

Figure 3.5: Moving towards equilibria. Case 1



Case 1. $0 < s < s_L$. Because this saving rate is smaller than the lower equilibrium saving rate s_L, the initial rise in productivity is less than W_l. Therefore, the economy will start from point A_3 in Figure 3.5, which is below point E_L. This starting point is located in region E, or B for this particular saving rate. Therefore, the productivity declines over time, but the efficient capital stock will increase, except

perhaps only for some initial periods. In the long-run, as the induced rise in productivity approaches zero, the economy will end up with a no-growth situation.

The reason for this can be explained as follows. Because the saving rate is relatively small, the initial rate of investment is not large enough for the economy to attain even the lower balanced growth path. As time passes, although investment still continues to take place, the rise in productivity induced by this investment activity is not large enough to offset the fall in productivity due to diminishing marginal productivity of capital. The output still grows but at a rate which becomes smaller and smaller than that of physical capital. Since the saving rate is fixed, as capital stock grows, even though investment is still positive, the proportional rate of growth of the capital stock eventually approaches zero. Therefore, the long-run growth rate equals zero.

It should be noted that this situation is similar to that in the traditional Solow model without growth in technology and the labour force. The only difference is that the process of moving toward a no-growth state in this model is slower, due to the offsetting, even small, effects of investment on the fall in the productivity of capital.

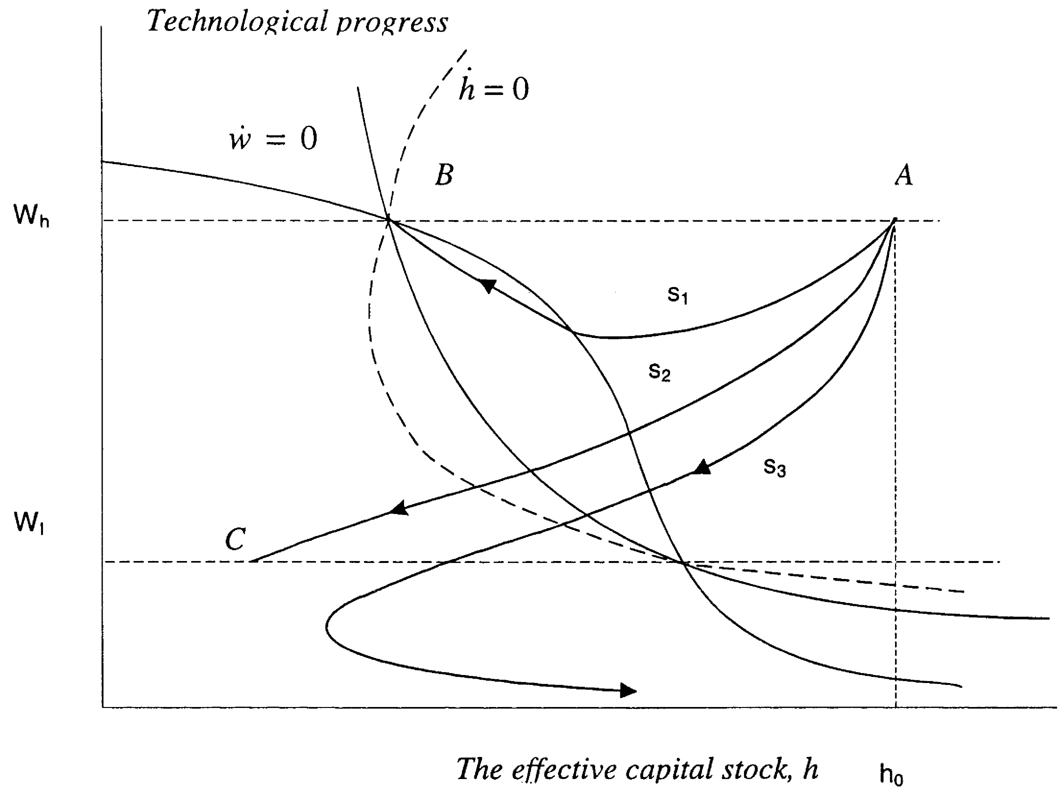
Case 2. $s_L < s < s_H$.. In this case, the economy will start at some point A_2, located between two equilibrium points. The initial rise in productivity is less than W_H, but bigger than W_l. This allows the economy to avoid the no-growth trap in Case 1, but it takes time to reach the higher growth level. Because during the transitional period, output grows faster that the physical capital stock, productivity increases, but the effective capital stock declines until the economy reaches its equilibrium point B_2.

Case 3 $s > s_H$.. In this case, the economy will start at some point A_1, which is above the higher equilibrium point. The initial rise in productivity is bigger than W_l. However, as in Case 1, except maybe for some initial periods, the rise in productivity induced by the investment activity cannot offset the fall in productivity due to diminishing marginal productivity of capital, because the rate of investment is too high. Output grows slower than the physical capital stock, productivity will decline, and the rate of investment will slow down until the economy reaches its equilibrium point B_1 with a higher efficient capital stock.

3.4.2 Effects of change in saving rate on growth: A graphical analysis

The second question about the effects of change in saving rate on growth, when the economy is already on a balanced growth path, can be considered in a similar manner to the previous section. Suppose the economy is at point A in Figure 3.6. The actual efficient capital stock is h_0, and the growth rate of productivity is W_h. A drop in the saving rate will reduce the investment for the next period. This results in a decline in the rate of growth of physical capital stock, so the growth in productivity induced by the investment also falls. Consequently, the rate of change in total productivity decreases and so does the efficient capital stock. How long this downward movement of the economy lasts will depend on the size of change in the saving rate as shown in Figure 3.6.

Figure 3.6: Moving towards equilibria. Case 2.



If the drop in saving rate is small enough, this downward trend can be reversed: after some time period, the economy will stabilize, the growth rate in output will take over the rate of growth of capital. Productivity will increase again, but the efficient capital stock continues to decline until the economy reaches a new equilibrium B, with a lower steady-state value of the efficient capital stock, but the long-run growth rate is back to the previous, pre-shock level. In this case, a change in saving rate has only temporary, but not long-run effects on growth.

Two other cases are also possible. There exists a saving rate, say s_2 which drives the economy to the lower balanced growth path. The economy ends up in the new equilibrium C with a lower level of the efficient capital stock and a lower long-run growth rate.

Finally, a second case where the economy may end up in the no-growth situation is also possible. This may occur if the change in saving rate is so significant that the downward trend in productivity cannot, by any means, be reversed. In the long-run the dynamics of the economy is similar to that in Case 1 in the previous section. The investment level is still positive, but its growth rate approaches zero and so does the growth rate of output.

The above analysis confirms the proposition about the relative sensitivity of the long-run growth rate with respect to the saving rate. There exist some threshold values of saving rate, passing through which may lead the economy to different steady-states with different long-run growth rates. The long-run growth rate, therefore, is not totally policy-invariant.

3.5 Conclusion

Modelling technological change and incorporating it into a growth model is important in constructing an endogenous growth model. In this model, we adopt different approaches to this issue. In the choice of variables for the technological progress function, the Romer approach has been used, so the rate of change in productivity induced by the investment activity is a function of the rate of growth in capital. Compared to the Romer model, this model considers *explicitly* the dynamics of physical capital stock and *implicitly* the dynamics of knowledge accumulation. It can be shown that in the reduced form, the two approaches have a similar derived production function. In this sense, the model constructed here may be considered as an extension of the Romer model in a specific way. Furthermore, the assumption about the S-shaped form of the technology-shock generating function leads to multiple equilibria. In this case, the long-run growth rate is no longer policy-neutral. In particular, different saving rates may lead the economy to different long-run growth paths.

CHAPTER 4

GROWTH AND WELFARE IN A MODEL WITH ENDOGENOUS TECHNOLOGICAL CHANGE.

In the previous chapter, I have shown that it is possible for an economy to have a perpetual positive long-run growth rate if productivity is affected by the investment activity. The model constructed so far is endogenous in the sense that the investment activity measured by the growth rate of physical capital stock can have not only level effects but also permanent effects on the long-run growth rate of the economy. The previous chapter analysed the growth paths resulting from the choice of any particular level of investment but left open the question about how this choice is made. This chapter looks at the choice of the level of investment, hence the choice of the appropriate growth paths made by an utility-maximising representative household in the economy. It will be shown that in the presence of multiple balanced growth paths, one of these paths is optimal and the economy may end up in the higher or lower growth path. Time-preference has not only level effects on welfare and ouput but may also play an important role in deciding which growth path, the higher or the lower, the economy will follow. Other things being equal, an impatient society, which is characterized by a higher discount rate would probably prefer the lower growth path. Alternatively, a more forward-looking society with a lower discount rate would prefer the higher growth path. The analysis also examines the effects of improvements in learning ability of the economy on growth. A surprising result is that although an improvement in learning ability is ultimately welfare-enhancing, its effects on long-run growth are ambiguous: the economy may grow faster or slower depending on which balanced growth path is preferred.

4.1 Balanced growth path and optimal saving rate

Recall that the economy in consideration is characterized by the following system of equations:

$$G(h) = h^{\alpha} \qquad (4.1)$$

$$\frac{dw}{dt} = \frac{g(\theta_t)}{1-\alpha} - \beta w \qquad (4.2)$$

$$\frac{\dot{h}}{h} = \theta_t - (w + \dot{w}t) \qquad (4.3)$$

$$\theta_t = \frac{dk}{kdt} \qquad (4.4)$$

where $w(t) = \dfrac{z(t)}{1-\alpha}$ can be considered as a modified technological progress function

$G(t) = \dfrac{F(t)}{e^{w(t).t}}$ is the output, measured in terms of the efficiency unit of labour

$h(t) = \dfrac{k(t)}{e^{w(t).t}}$ is the capital stock, measured in terms of the efficiency unit of labour, and

$\theta(t)$ is the growth rate of the physical capital stock

Recall that Equation (4.1) stems from the underlying Cobb-Douglas production function with α being the share of capital in total output. Equation (4.2) relates the change in productivity to the growth rate of physical capital stock and finally, Equation (4.3) describes the dynamics of the effective capital stock during a capital accumulation process.

As in most growth models, we assume that the objective of a representative household is to maximize the discounted future flows of utility from consumption by, $u(C_t)$, that is the overall utility is given by:

$$U = \int_0^\infty e^{-\rho t} u(C_t) dt$$

For simplicity, consider the case where $u(C_t) = ln\,(C_t)$

Let $c(t)$ be the consumption, measured in terms of the efficiency unit of labour Then $C(t) = e^{wt} c(t)$. Furthermore, it can be verified that the relationship between consumption c, the rate of growth of *physical capital*, θ and the effective capital stock h is given by:

$$c = h^\alpha - \theta h$$

Therefore, the optimization problem can be rewritten in terms of transformed variables as follows:

$$\max \int_0^\infty e^{-\rho t} (wt + \ln c) dt \qquad (4.5)$$

subject to:

$$\dot{w} = \frac{g\left(h^{\alpha-1} - ch^{-1}\right)}{1-\alpha} - \beta w \qquad (4.6)$$

$$\dot{h} = h^\alpha - c - h(w + \dot{w}t) \qquad (4.7)$$

Two difficulties are encountered in dealing with this dynamic optimizing problem. Firstly, the system is non-autonomous due to Equations (4.5) and (4.7), and secondly, the feasible set as defined by Equations (4.6) - (4.7) is not necessarily convex due to the non-concavity of function *g(.)* in Equation (4.6). These properties of the system make it very difficult to solve the problem explicitly by using the standard dynamic optimizing technique. The non-autonomous property complicates the possible time-paths of the main variables, and it is possible that the notion of steady-state can only be usefully applied in a more extended version. Secondly, the non-convexity property cannot provide the sufficient conditions for optimality. A direct application of the standard dynamic optimizing technique does not lead to the desired results (see Appendix 4.A). Therefore, we need to adopt a more extended definition of steady-state, in order to capture the complexity of the non-autonomous system, and to deal with the non-convexity of the problem. The latter turns out to be solvable if we consider the growth rate of physical capital stock as a control variable, instead of using the consumption variable. Now, all main state variables are considered according to their relationship with this newly introduced control variable.

Suppose, for simplicity, that the capital accumulation process is given by a simple sequence $\{\theta(t) = \theta\}$ for all t. The modified technological progress function w(t) can be found explicitly, by solving the corresponding differential equation in (4.2). It yields:

$$w(t) = \frac{g(\theta)}{(1-\alpha)\beta}(1 - e^{-\beta t}) \quad (4.8)$$

and $\dot{w}(t) = \dfrac{g(\theta)e^{-\beta t}}{(1-\alpha)}$ for all t>0

The evolution of the capital stock per capita, measured in terms of the efficiency of labour *h(t)* is characterised by the following differential equation:

$$\frac{\dot{h}}{h} = \theta - (w + \dot{w}t) = \left[\theta - \frac{g(\theta)}{(1-\alpha)\beta}\right] + \frac{g(\theta)e^{-\beta t}}{(1-\alpha)\beta}(1-\beta t) \qquad (4.9)$$

Definition:

A triple (θ^*, w^*, h^*) represents an *asymptotic* steady-state for the system (4.2)-(4.4) if there exists a growth path (θ_t, w_t, h_t) such as :

$$\lim_{t\to\infty} \theta_t = \theta^* \qquad \lim_{t\to\infty} w_t = w^* \quad \text{and} \quad \lim_{t\to\infty} h_t = h^*$$

It can easily be seen that the asymptotic steady value of the technological parameter w^* and that of the efficient labour h^* are the functions of the corresponding steady value of the growth rate of the capital stock, that is

$w^* = w(\theta^*)$ and $h^* = h(\theta^*)$

Furthermore, these two functions have a quite interesting property, which can be expressed in the following Remark 4.1

Remark 4.1. *The asymptotic steady values of the technological parameter* $w(\theta)$ *and of the efficient labour* $h(\theta)$ *as functions of the growth rate of the capital stock* θ, *are given by*

$$w(\theta) = \frac{g(\theta)}{(1-\alpha)\beta} \quad \textit{and},$$

$$h(\theta) = \begin{cases} > 0 & \text{if } \theta\beta(1-\alpha) = \mathrm{g}(\theta) \\ \infty & \text{if } \theta\beta(1-\alpha) > \mathrm{g}(\theta) \\ 0 & \text{if } \theta\beta(1-\alpha) < \mathrm{g}(\theta) \end{cases}$$

It is important to note that while the function $w(\theta)$ is continuous in θ, the function $h(\theta)$ is not. Given the shape of the technology-shock generating function $g(.)$ as in the previous chapter, two jumps occur for the function $h(\theta)$ at two boundary points associated with the two balanced growth paths with the rate of balanced growth being equal θ_1 and θ_2,. Outside the interval [θ_1, θ_2], the effective capital stock tends to infinity because the growth rate of the physical capital stock is greater than the increase in productivity induced by the investment process. Inside the interval [θ_1, θ_2], however, the effective capital stock eventually tends to zero. This discontinuity of the function $h(\theta)$ has a very important implication for the optimal solution to the intertemporal utility-maximising problem: it reinforces the possibility that the optimal solution is at one of the two boundary points.

The optimization problem, in terms of transformed variables, can be rewritten as follows:

$$\max \int_0^{\infty} e^{-\rho t}(wt + \ln(h^{\alpha} - \theta h)dt \qquad (4.10)$$

$$\dot{w} = \frac{g(\theta)}{1-\alpha} - \beta w \qquad (4.11)$$

$$\dot{h} = \theta h - h(w + \dot{w}t) \qquad (4.12)$$

$$\theta_1 \le \theta \le \theta_2 \text{ or } \frac{g(\theta)}{(1-\alpha)\beta} \ge \theta \qquad (4.13)$$

This problem can be written in another more convenient form by introducing a new variable

$v = wt.$

The system (4.10)-(4.13) now becomes:

$$\max \int_0^\infty e^{-\rho t}(v + \ln(h^\alpha - \theta h)dt \qquad (4.14)$$

$$\dot{v} = \frac{g(\theta)t}{1-\alpha} - \beta v + \frac{v}{t} \qquad (4.15)$$

$$\dot{h} = h(\theta - \dot{v}) \qquad (4.16)$$

$$\theta_1 \le \theta \le \theta_2 \qquad (4.17)$$

It should be noted that the objective function contains two components, each of which has its own interpretation. Let

$$J = \int_0^\infty e^{-\rho t}(v + \ln(h^\alpha - \theta h)dt$$

$$J_1 = \int_0^\infty e^{-\rho t} v dt \text{ and } J_2 = \int_0^\infty e^{-\rho t} \ln(h^\alpha - \theta h)dt$$

The first component J_1 which reflects the welfare effects of an improvement in productivity due to investment activity can be considered as a growth effect on consumption. The second component J_2 can be considered as a level effect. The relative and absolute size of these two components are affected by the discount rate and the growth rate of the physical capital stock, but in different ways. This in turn will affect the choice by the economy between different balanced growth paths as will be shown later.

The Hamiltonian is given by

$$H = v + \ln(h^{\alpha} - \theta h) + \mu\left(\frac{g(\theta)t}{1-\alpha} - \beta v + \frac{v}{t}\right) + \lambda h\left(\theta - \frac{g(\theta)t}{1-\alpha} + \beta v - \frac{v}{t}\right)$$

(4.18)

The first-order conditions for optimality to this problem can be found by using the standard dynamic optimization technique and are given by the following system of equations:

$$\frac{\partial H}{\partial \theta} = -\frac{h}{h^{\alpha} - \theta h} + \lambda h + \frac{g'(\theta)t}{1-\alpha}(\mu - \lambda h) = \begin{cases} 0 \text{ if } \theta_1 < \theta < \theta_2 \\ > 0 \text{ if } \theta = \theta_2 \text{ and } < 0 \text{ if } \theta = \theta_1 \end{cases} \tag{4.19}$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial w} = \rho\mu + (\mu - \lambda h)(\beta - \frac{1}{t}) - 1 \tag{4.20}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial h} = \rho\lambda - \frac{\alpha h^{\alpha-1} - \theta}{h^{\alpha} - \theta h} - \lambda(\theta - \dot{v}) \tag{4.21}$$

Proposition: *For an economy described by the system of Equations (4.14)-(4.17), one of the two accumulation paths associated with two balanced growth paths of the capital stock θ_1 and θ_2 is optimal.*

Proof: Recall from Chapter 3 that a balanced growth path is associated with the long-run growth rate of the physical capital θ, which is defined from the condition: $\theta\beta = \dfrac{g(\theta)}{1-\alpha}$

Given this value of the growth rate in capital , the new productivity variable v can be found explicitly by solving the differential equation (4.15). It yields a simple functional form

$$v(t) = \frac{g(\theta)t}{(1-\alpha)\beta}$$

Along this accumulation path, the effective capital stock h remains unchanged; hence it attains a steady-state, since, by assumption $g(\theta) = (1-\alpha)\beta\theta$. Thus

$h(t) = h^*$ for all t

We will show now that there exist time-functions $\mu(t)$ and $\lambda(t)$ such that the quintuple $(\theta, v, h, \mu, \lambda)$ satisfies the FOCs for optimality.

Let : $\alpha h^{\alpha-1} = \theta + \rho$ (4.22)

and $\lambda = \dfrac{1}{h^{\alpha} - \theta h}$ (4.23)

Substituting (4.22) and (4.23) into (4.20) yields a differential equation in which $\mu(t)$ is a function of time only. Using the standard method of solving a linear differential equation yields the following explicit solution :

$$\mu(t) = \frac{Me^{(\rho+\beta)t}}{t} + \frac{1-\rho\lambda h}{\rho+\beta} + \frac{1-\rho\lambda h}{(\rho+\beta)^2 t} + \lambda h$$

(For more details, see Appendix A)

If M is large enough , then $\mu - \lambda h > 0$ for all t . It implies that $\frac{\partial H}{\partial \theta} > 0$ .In this case, the balanced growth path associated with *higher* θ will satisfy all FOCs necessary conditions for optimality. In the case of a concave dynamic optimization problem. the necessary conditions similar to (4-19)- (4-21) would also be the sufficient ones and the optimal solution would be unique. However, due to the non-convexity of the technology-shock generating function *g(.)* and the discontinuity property of the effective capital stock as a function of the growth rate of physical capital, it is possible that even the lower balanced growth path is optimal because of a jump at these two boundary points.

To show that only the two corner solutions associated with the two balanced growth paths may be optimal, it is necessary to show that any interior growth path that is $\theta_1 < \theta < \theta_2$ cannot be optimal. For this purpose, we apply a well-known approach in finding an optimal growth path, adopted by Rebelo (1991) Rivera-Batiz and Romer (1993), Barro and Sala-i-Martin (1992, 1995) and Lee (1995). According to this approach, the conditions for an optimal growth equilibrium come from the conditions of equilibrium in both the production sector, i.e from the producer's point of view as well as from preferences, i.e. from the consumer's point of view. Given the utility function u(C) = ln(C), for any growth rate of consumption $g_C = \dot{C} / C$, the implied interest rate for the consumer can be calculated as follows:

$$r_c = \rho + g_c.$$

We may view r_c as the effective discount rate which is the required premium in future consumption over current consumption (Barro and Sala-i-Martin, 1992). The condition reflects a positive relation between the interest rate and the growth rate of consumption because when consumption is growing more rapidly, current consumption is more valuable compared to future consumption, so the marginal rate of substitution between present and future consumption is higher. Consumers are willing to borrow at a higher interest rate.

On the other hand, from the production side, the producer is willing to pay an interest rate r_p equal to the marginal return to capital . This implies that

$$r_p = \frac{\partial F}{\partial K}$$

The equilibrium condition in the capital market, therefore, requires

$$r_c = \rho + g_c = r_p = \frac{\partial F}{\partial K}$$

This condition can serve as an implicit necessary condition for an optimal growth path.

Now we can use this criterion to show that none of the interior growth paths can be optimal. Recall from Remark 4.1 that if the capital stock grows at a constant rate θ, it will result in a steady value of technological parameter equal to $w = \frac{g(\theta)}{(1-\alpha)\beta}$. Therefore, the output will grow at a rate $\gamma = \frac{g(\theta)}{\beta} + \alpha\theta$. Since θ is an interior value, this growth rate of output is larger than the growth rate of the capital stock θ. Consumption, therefore, grows at the same rate as the output. The implied interest rate for consumer is given by

$$r_c = \rho + g_c = \frac{g(\theta)}{\beta} + \alpha\theta + \rho$$

On the other hand, since the output is given by $F(t) = F(0)e^{\gamma t}$ and the capital stock is $K(t) = K(0)e^{\theta t}$, the marginal product of capital can be calculated as follows

$$\frac{\partial F}{\partial K} = e^{wt}\alpha K_t^{\alpha-1} = \alpha K(0)^{\alpha-1} e^{(w+(\alpha-1)\theta)t} = \alpha K(0)^{\alpha-1} e^{(\gamma-\theta)t}$$

The induced long-run production function, which relates the output to the capital stock as functions of time, therefore, exhibits increasing returns to capital. As t approaches infinity, the marginal product of capital also tends to infinity. The would-be interest rate for the producer, therefore, grows at a rate equal to the difference between the growth rate of output γ and the growth rate of capital θ. This means that as the economy grows, the productivity of capital also increases, the producer is willing to pay a higher interest rate in order to secure more capital. However, the implied interest rate for the consumer remain unchanged. The capital market could never be in equilibrium. The growth path, therefore, is not optimal.

As a matter of fact, because the productivity of capital increases, it puts pressure on the interest rate for the consumer because a higher interest rate would be required to induce households to postpone current consumption and to consume later, and hence to save more. This can be done only by an increase in the growth rate of capital stock until it reaches the higher boundary defined by the higher balanced growth rate. The economy has a tendency to move towards a higher growth path. The higher growth path, therefore, is stable as has been shown in Chapter 3.

What about the lower growth path? As in the case of the higher balanced growth path, the lower growth path can also be associated with equilibrium conditions for the capital market. The only difference is that the lower growth rate results in a lower effective discounted rate (or a lower implied interest rate for consumers) and hence requires a higher steady value of the effective capital stock, as defined in (4.22). The lower growth path, therefore, may also be optimal

It is necessary to note that although one of the balanced growth path may satisfy the necessary first-order conditions for optimality, this cannot guarantee that these growth paths are optimal, because owing to the convexity of the objective function, the first-order conditions for optimality are not sufficient. Which one of the 'corner' solutions is optimal depends on the values of the objective welfare function. The latter can be calculated as follows:

$$V(\theta) = \int_0^\infty e^{-\rho t}(v + \ln(h^\alpha - \theta h)dt$$

$$= \int_0^\infty e^{-\rho t}(\theta t + \ln(h^\alpha - \theta h)dt = \int_0^\infty e^{-\rho t}(\theta t + \ln h + \ln(h^{\alpha-1} - \theta)dt$$

$$= \int_0^\infty e^{-\rho t}(\theta t + \frac{\ln(\theta+\rho) - \ln\alpha}{\alpha - 1} + \ln[(1-\alpha)\theta + \rho] - \ln\alpha)dt$$

$$= \int_0^\infty e^{-\rho t}(\theta t + -\frac{\ln(\theta+\rho)}{1-\alpha} + \ln[(1-\alpha)\theta + \rho] + \frac{\alpha\ln\alpha}{1-\alpha})dt$$

$$= \frac{1}{\rho^2}(V_1 + \rho V_2)$$

where $V_I = \theta$ and $V_2 = \ln[(1-\alpha)\theta + \rho] + \frac{\alpha\ln\alpha}{1-\alpha} - \frac{\ln(\theta+\rho)}{1-\alpha}$

It is clear now that the effects of the rate of growth of capital on the two components in the welfare function V_1 and V_2 are explicitly given by

$$\frac{\partial V_1}{\partial \theta} = 1 > 0 \text{ and } \frac{\partial V_2}{\partial \theta} = \frac{1-\alpha}{(1-\alpha)\theta+\rho} - \frac{1}{(1-\alpha)(\theta+\rho)} < \frac{-\alpha}{(1-\alpha)(\theta+\rho)} < 0$$

Therefore, there is a trade-off for the economy in choosing the lower and the higher growth paths. The higher growth path generates higher growth effects on the welfare function because $\frac{\partial V_1}{\partial \theta} > 0$, while the lower path results in a high level effect since $\frac{\partial V_2}{\partial \theta} < 0$. Given this possible trade-off and the relative size of the two components, the ultimate decision as to which growth path to follow depends on the discount rate ρ. An impatient society, which is characterized by a higher discount rate would probably prefer the lower growth path. By contrast, a more forward-looking society with a lower discount rate would prefer the higher growth path. This feature distinguishes the model from the traditional Solow model, where the time-preference has no effect on the long-run growth rate because the latter is exogenously given.

It is important to note that regardless of which balanced growth path is chosen, the steady-state effective capital stock and the steady-state saving rate in both cases have the same form and are given by

$$\alpha \bar{h}^{\alpha-1} = \theta + \rho \qquad (4.24)$$

and $$\bar{s} = \frac{\theta\alpha}{\theta+\rho} \qquad (4.25)$$

where $\bar{h}$ is the steady-state effective capital stock and,

$\bar{s}$ is the steady-state saving rate

In Equation (4.24) the left-hand side expression is the marginal productivity of the effective capital stock. It is not difficult to show that it also represents the marginal productivity of the physical stock, since

$$\frac{dF}{dK} = \frac{\alpha F}{K} = \frac{\alpha G}{h} = \alpha h^{\alpha-1}$$

The right-hand side of this equation is the effective discount rate, which consists of the discount rate and the effects from diminishing marginal utility of consumption due to growth of consumption at the rate θ equal to the rate of growth of the physical capital stock (Barro and Sala-i- Martin, 1995). Therefore, Equation (4.24) simply says that, at the equilibrium, the marginal productivity of capital equals the effective discount rate. This condition is consistent with implications drawn from most growth models and the traditional Solow model, in particular.

Equation (4.25) expresses the relationship between the steady-state optimal saving rate and the share of capital in total output α , the discount rate ρ, as well as the optimal growth rate θ. Once again this formula is the same as in other growth models. The only difference here is that the optimal growth rate is defined within the model, rather that being given exogenously.

4.2 Effects of learning ability on growth and saving: A comparative static analysis

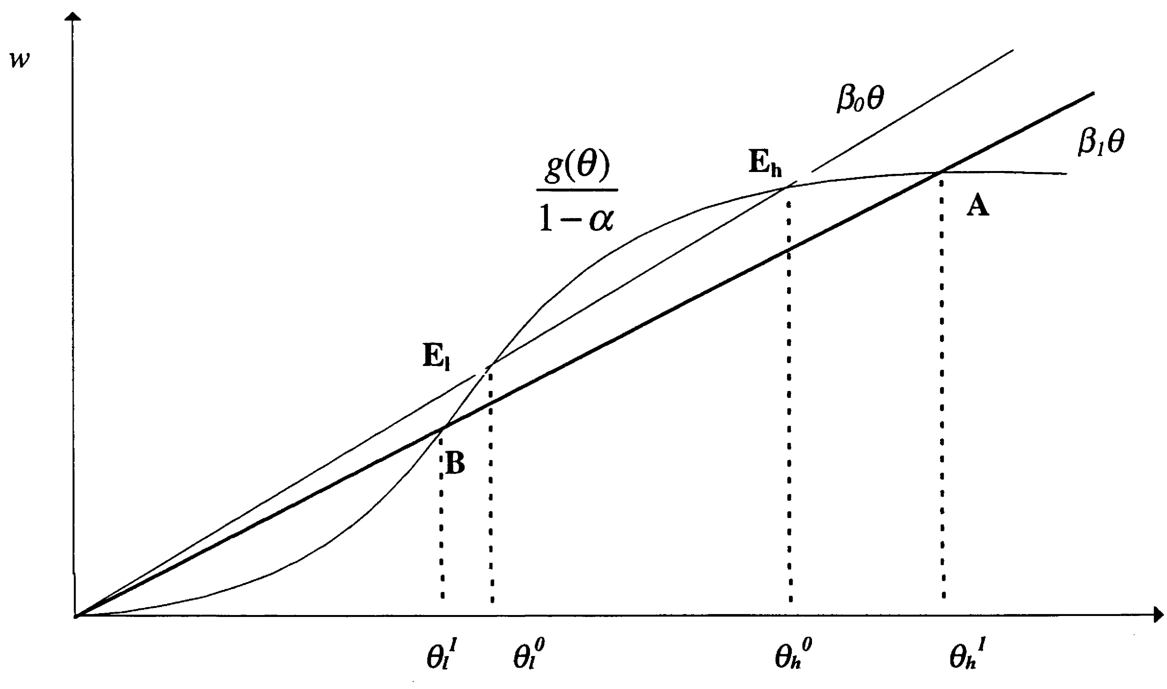
In this section, we will examine the possible effects of changes in learning ability on long-run growth and other related variables such as the steady-state saving rate and the steady-state effective capital stock. By a change in learning ability we understand any change which could affect the degree of perfection in the knowledge transmission process, β, or which could result in a shift in the technology-shock generating function, $g(\theta)$. Since these two kinds of change are equivalent in the sense that their final outcome is to change the growth rate of productivity, we can restrict our consideration by looking only at the effects of changes in β.

From Figure 4.1, it can be seen that an improvement in the knowledge transmission process (β becomes smaller) results in a widening of the range of feasible growth rates of capital , because the lower-bound of θ declines while the upper-bound increases. However, the effects of this improvement in learning ability on the optimal long-run growth path are different, depending on whether the economy is at the lower or higher equilibrium.

a) Higher Equilibrium Case.

From Figure 4.1 it can be seen that a decline in β from β_0 to β_1 results in an increase in the long-run growth rate from θ_h^0 to θ_h^1. The corresponding steady-state of capital stock also increases. It would require a higher saving rate as given in (4.25). Therefore, the main variables will move in the same direction with the learning ability. This can be explained by the fact that the economy is forward-looking (because it has chosen the higher growth path). Thus, even though an improvement in the learning ability would provide a better opportunity for the economy to improve its welfare (and consumption) without changing the rate of growth of investment, the economy is not satisfied with this option. Rather, due to its forward-looking nature, the economy is better off to save more and to speed up the rate of growth of capital in seeking more consumption in the future. As a result, the long-run growth rate increases.

Figure 4.1 Learning Ability and Long-Run Growth



<u>*b) Lower Equilibrium Paradox?.*</u>

The change in long-run growth due to an improvement in learning ability is different if the economy started from the lower equilibrium level. As shown in Figure 4.1, a decline in β from β_0 to β_1 results in a fall in the long-run growth rate from θ_l^0 to θ_l^1. The corresponding steady-state of capital stock also falls . So does the saving rate. The main variables will move in the opposite direction from the learning ability. This apparent growth paradox is not difficult to explain if one recalls that the economy is at the lower equilibrium because of its impatience and hence prefers present consumption to that in the future. Although, an improvement in learning ability does provide a better opportunity for the economy to improve its productivity and therefore, to generate more consumption flows in the future, the economy is safe to devote fewer resources to investment without fear of significantly slowing down the future flows of consumption. The welfare is improved even if the economy is willing to save less. Therefore, an increase in learning ability is always welfare-enhancing.

It should be noted that this apparent paradox is not an exception. A similar phenomenon can be observed in the labour market when we consider the work-leisure choice of a worker. An increase in wage rate, in this case does not necessarily lead to an increase in work-hours by the worker. The worker may work less or more depending on his or her relative preference between consumption and leisure. If the substitution effect dominates then he/she will devote more time to work. Otherwise, the person will work less. But in both cases, his/her overall utility will definitely increase.

4.3 Transitional dynamics

We have established so far that the economy may attain the maximal value of the welfare function in one of the two balanced growth paths. The next question is how the economy can move towards this equilibrium given a set of initial conditions such as the initial stock of physical capital and hence that of the effective capital stock. This problem involves transitional dynamics which is considered intensively and thoroughly by Barro and Sala-i-Martin (1995) for many types of growth models.

In this section we will apply the Barro and Sala-i-Martin technique to this particular model.

To begin with, it is worthwhile to note that the initial efficient capital stock h_0 generally differs from the steady-state optimal value h^*. The latter may be higher or lower than the former depending on values of the discounted rate ρ , the long-run optimal growth rate θ as well as the share of the physical capital stock in total output as shown in Equation (4.24). Intuitively, one may suggest that if the initial capital stock is lower than the optimal level, it is necessary to increase the capital stock at a rate higher than that of the long-run rate during the transitional period. However, due to different stability properties of two balanced growth paths, the adjustment path for these two cases is quite different as will be shown later. In what follows, we will consider these two cases separately.

Case 1: Transitional Dynamics towards the higher growth path.

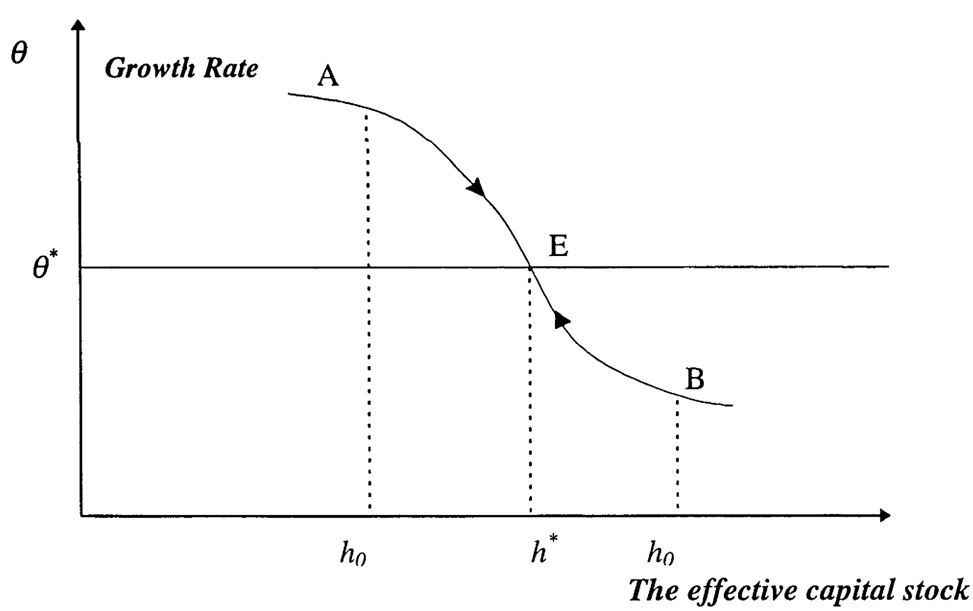
Suppose the economy starts with the capital stock equal to h_0 and the optimal level is h^* and $h_0 < h^*$ as in Figure 4.2. The optimal saving ratio required for this higher growth path is defined by

$$s^* = \frac{\theta\alpha}{\theta + \rho}$$

In order to increase the efficient capital stock, the growth rate of physical capital stock must be higher than that in the long-run, because only in this case can the increase in the physical capital stock exceed the induced increase in the productivity, so the effective capital can reach the optimal level. Therefore, during the transitional period, the saving rate remains constant and equals the long-run rate. As a result, the growth rate of physical capital stock gradually slows down until it reaches the optimal level. The economy moves from the initial point A to the equilibrium point E as shown in Figure 4.2 .

A similar adjustment would be required in the case where the initial level of the effective capital stock is higher than the equilibrium level. As before, the saving ratio remains constant during the transitional period. However, the growth rate of physical capital stock gradually increases. The economy is moving from B to point E in Figure 4.2.

Figure 4.2- Transitional Dynamics: The Higher Equilibrium Case



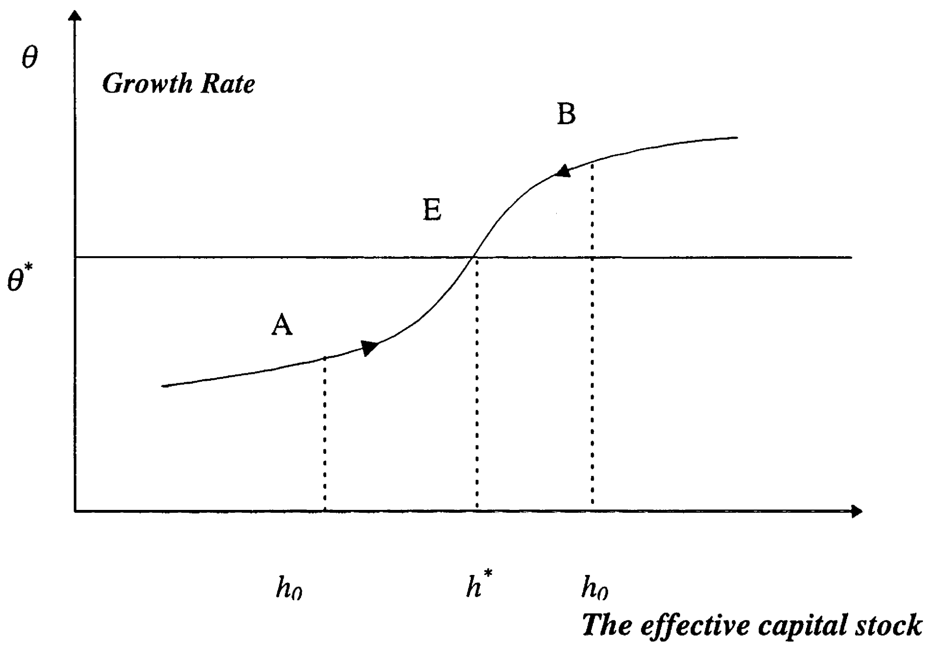
One common feature for the adjustment process in both cases is that there is no change in the saving ratio. It results automatically in an appropriate change in the growth rate of the physical capital stock in order to attain the desired level of the effective capital stock. This constancy in the saving rate property can be explained by the stability of this higher growth equilibrium. As has been shown in the previous chapter, a wide range of constant saving rates will lead to the same high level of growth in the long run.

Case 2: Transitional Dynamics towards the lower growth path.

As shown in the previous chapter, the lower balanced growth path is unstable in the sense that any one-off change in the saving ratio can lead the economy to a long-run growth pattern which is different from the original one. This instability property requires a special adjustment process if the economy is to attain this lower equilibrium from an initial capital stock different from the optimal level. In this case, reaching the equilibrium requires both the saving rate and the growth rate of the physical capital stock to adjust. If the initial capital stock is lower than the desired level, the saving rate must start from a value lower than that in the long-run but must increase in such a manner that the resulting growth rate of capital also gradually increases to the steady-state level. By contrast, if the initial capital stock is higher than the desired level, both the saving rate and the resulting growth rate of capital must gradually move downward to the equilibrium levels. This adjustment process is depicted in Figure 4.3. The economy may move from initial point A to the equilibrium E if the initial level of the capital stock is lower than the optimal one. Alternatively, the economy would move from point B to E with both saving rate and the growth rate of the physical capital higher from the start but gradually approaching the corresponding values in the steady-state.

In summary, the following policy rules are applied to the economy during its transition to the balanced growth paths.

Figure 4.3- Transitional Dynamics: The Lower Equilibrium Case



During the transitional period towards the higher optimal balanced path, the saving ratio is constant and equal to the optimal level. This constancy in the saving ratio will automatically drive the rate of growth in the capital stock to the optimal level. This movement of the rate of growth in the physical capital stock is downward, if the initial level of the effective capital stock is lower than the long-run level. Otherwise, the movement is upward. The transition to the lower optimal balanced path requires a more complicated adjustment process of the saving rate. The direction of movement of the rate of growth in capital stock is opposite to that in the case of the higher balanced growth path. That is, if the initial capital stock is higher (lower) than the optimal level, then the saving rate is moving downwards (upwards) to the equilibrium.

CHAPTER 5

A GROWTH MODEL FOR AN OPEN ECONOMY:

IMPORTED CAPITAL GOODS AND LONG-RUN GROWTH

5.1 Motivation

Growth models for an open economy have been analysed in different frameworks. Before the new growth theory emerged, the main focus was placed on welfare and transitional analysis, as well as the transmission mechanism due to external policy such as taxation and trade policies. Bardhan (1967), for example, considers a simple model of foreign borrowing . The main objective of his paper is to investigate the effects of borrowing on the welfare of the economy and to determine the optimum level of borrowing. Pitchford (1994) extends this analysis to a very wide range of issues such as government borrowing, foreign investment, borrowing or investing with adjustment costs, borrowing for consumption or investment. This kind of analysis provides useful insights into some very important policy questions such as the optimal level of public and/or private borrowing and the need for government intervention into the capital market. However, all these analyses have been carried out in terms of levels. In this sense, they are all very much traditional: the international capital market or the opportunity to borrow has nothing to do with the long-run growth rate. Pitchford in one of his models, considers the external indirect effects of foreign investment and argues that the stock of foreign capital can increase the efficiency of labour because foreign investment ' might employ technology or products not available in the country concerned or it might enhance the productivity of the labour force who learn new skills from 'on the job' experience which they can use widely' (Pitchford (1994)). However, even within his extended model, the growth rate remains exogenous for one simple reason: the specification is very similar to Arrow's model.

Barro and Mankiw (1992) attempted to construct a model of economic growth that is, as the authors argue, consistent with the growing body of evidence on convergence. The capital is partially mobile: borrowing is possible to finance the accumulation of physical capital but not the accumulation of human capital. The opportunity to borrow abroad or the degree of openness in the capital market does not influence the steady-state, let alone the long-run growth rate. It may only affect the speed of convergence. The model, therefore, is more transitional than long-run. Furthermore, as Barro admits, the model is best applied not to countries or even to states, but to families and may be useful for explaining the dynamics and distribution of wealth (Barro and Mankiw, 1992)

Turnovsky and Brock (1993) examine the transitional dynamic adjustment path following a tax increase in a country. The conclusion is that the tax increase leads to an initial transfer of capital from the domestic to a foreign country and also to a decline of the world capital stock. Lee (1993) considers the links between trade distortions and growth. The starting point is the premise that international trade can serve as a vehicle for providing foreign inputs which are important for production, because they are more efficient, especially for developing countries; therefore any trade restrictions on the availability of these foreign inputs and capital goods may hurt the economy. The model predicts that trade distortions can lower growth significantly over a long transitional period, because they impede the supplies of imported inputs, therefore reducing the productivity of capital accumulation.

Furthermore, the trade distortions have more serious repercussions for growth in small, resource-scarce countries than in large, resource-abundant ones. The capital tends to flow out from highly distorted, low-income countries to high-income countries with low levels of trade distortions.

It should be noted that although the above mentioned growth models for an open economy have interesting policy implications, all remain very much traditional: the focus has been placed on the transitional rather than on the long-run analysis, and the long-run growth rate remains totally exogenous.

New growth theory provides a new impulse to the study of growth in an open-economy context. Different ideas of endogenous growth have been captured in some open-economy models by Rivera-Batiz and Romer (1991), Romer (1993) Jong-Wha-Lee (1993, 1994), Easterly (1993).

Rivera-Batiz and Romer (1991), for example, examine the scale effects of economic integration on long-run growth. The authors consider the integration in three forms: trade in goods, flow of ideas and both trade in goods and exchanges of ideas between two countries with the same endowments and technologies. The research and development activity which aims to create new designs for new capital goods is considered in two different specifications: the knowledge-driven and the lab equipment specification. The first, knowledge-driven specification assumes that human capital and knowledge are the only inputs for producing new designs, that is

$$\dot{A} = \delta H A$$

where A is the existing stock of designs (knowledge)

and H is the stock of human capital used in research.

In the second, lab equipment specification, the research sector has a similar technology to the manufacturing sector: human capital, unskilled labor and capital goods, but not total knowledge, are essential to the research and development activity . Access to the designs for all previous goods and familiarity with the ideas and know-how does not aid the creation of new designs. The knowledge-creation function has the following form:

$$\dot{A} = BH^{\alpha} L^{\beta} \int_{0}^{A} x(i)^{1-\alpha-\beta}\, di$$

where, as before, A is the existing stock of designs (knowledge)

H is the stock of human capital used in research.

L is the amount of unskilled labor used in research.

$x(i)$ is the stock of capital type i used in research

The authors showed that with the knowledge-driven model of research, opening trade in (capital) goods has no permanent effect on the rate of growth. In balanced growth, the growth rate of output is determined by the relative allocation of human capital between the two competing, manufacturing and research sectors. After trade is opened, the number of types of machines used in each country almost doubles. The marginal productivity of human capital in both sectors, the manufacturing and the research, also doubles. However, the relative returns of human capital in the two sectors remain the same as before the opening of trade. The relative allocation of human capital between the two sectors, therefore, remains unaffected and so does the long-run growth rate.

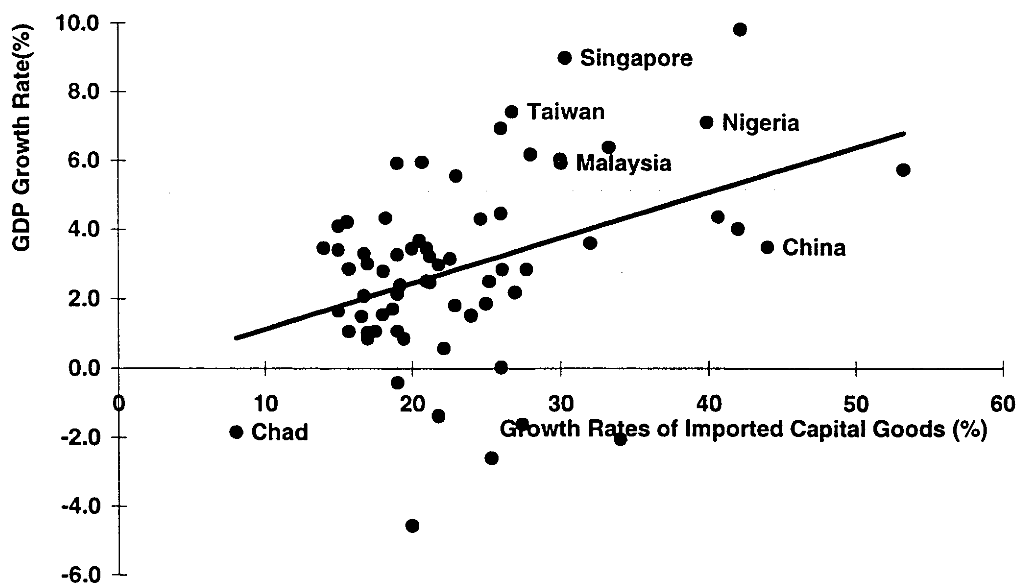
The same analysis can be used to consider the effects of integration on growth in other specifications. The authors came to the following conclusions:

Under the knowledge driven specification, allowing flows of ideas results in a permanently higher growth rate.

Under the lab equipment specification of the research sector, trade in goods alone causes the same permanent increase in the rate of growth as complete integration. However, since ideas per services have no effect on production, the creation of communication networks has no additional effect on growth.

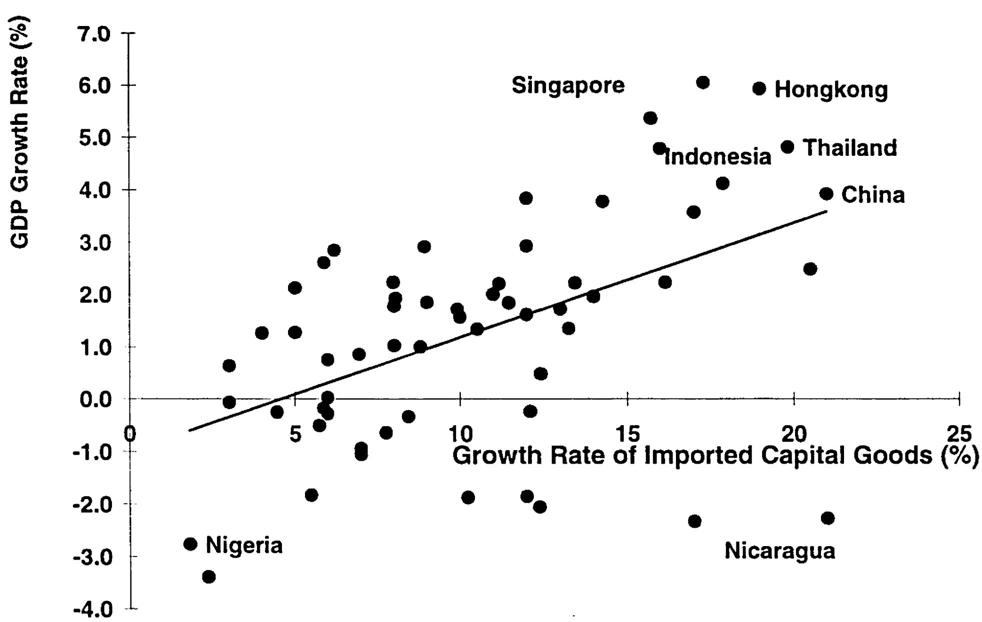
The model, as the authors admit, is most likely to apply to integration between similar developed regions of the world, such as, between North America, Europe and Japan. For less developed countries, Easterly (1993) and Lee (1994) proposed a simple model of an open economy by extending the Rebelo AK endogenous model. A less developed economy can grow faster if it can make use of cheaper imported capital goods. Unlike the previous version of Lee's model discussed before, the effects of international trade in capital goods on growth are permanent. The evidence seems to support this conclusion. Lee (1994) found that the ratio of imported to domestically produced capital goods in the composition of investment had a significant positive effect on per capita income growth rates across countries, in particular, in developing countries. Using the cross-section data of the 89 sample countries, he found that for a given value of initial income, school enrolment, population growth, and investment rate, countries grow faster if they use more imported capital goods than domestic capital goods in building their capital stock: an increase of 0.1 in the ratio of imports in investment leads to an increase in the growth of per capita income by 0.3 percent per year.

Figure 5.1 - Growth and Imported Capital Goods, 1970-80



Source: IEDB Database, and Summer and Heston PenTable 1995

Figure 5.2 - Growth and Imported Capital Goods, 1980-90



Source: IEDB Database, and Summer and Huston PenTable 1995

The possible relationship between imported capital goods and long-run growth rate can be viewed from a different perspective. Foreign inputs are important for developing countries because they bring with them new knowledge, new know-how.

The rate of growth of imported capital goods, therefore, can be seen as a technology-enhancing factor.

Figures 5.1 and 5.2 show the partial association between the growth rate of GDP and the growth rate of imported capital goods for a sample of countries in the last two decades: the 1970s and the 1980s. The growth rates (of GDP and of imported capital goods) are the average annual growth rates over the corresponding periods. Data on GDP are from Summers and Heston (1995), while data on imports of capital goods are extracted from the International Economic Data Bank (IEDB) at the Australian National University. The latter reports total value of imported machinery and transport equipment. The data show a significant positive correlation between the growth performance and the growth rate of imported capital goods. Most 'success stories' such as China, Singapore, Malaysia, Hongkong, South Korea are countries with the highest rate of growth of imported capital goods. However, the data cannot provide an answer to the question about the possible direction in the causal link between these two indicators. In some cases, it is possible that the high growth rate of imported capital goods is a result, not a cause of the high GDP growth rate. Nigeria, may be a good example. During the 70s, Nigeria was a big beneficiary of the terms-of-trade shock in 1973-1974. Being a big oil-exporting country with the oil production accounting for 17% of GDP and crude oil exports making up 85% of total export (Little *et al.*, 1993), Nigeria enjoyed quite a high growth rate during that period. A steady cash flow from the oil allowed the country to import more foreign inputs for other sectors, especially for the public development and capital expenditure in agriculture and rural development. However, during the next decade, the country experienced a serious setback and could no longer afford more imports. The slow-down in the overall economic performance, led to a decline in imports. From 1981 to 1983, oil exports were cut by half, investment collapsed, import licensing become more restrictive, the country was depressed and suffered from import starvation (Little *et al.*, 1993).

However, there exists a long-held conviction among development economists, especially among advocates of direct foreign investment, that the relationship may well be in the opposite direction; that is importing more foreign 'superior' inputs may facilitate the development process. East-Asian countries are often cited as an example. Indonesia, for instance, also benefited from the oil shock in the 1970s. However, unlike Nigeria, Indonesia succeeded in avoiding the Dutch disease: the oil boom did not squeeze out other exporting sectors. By contrast, using the oil revenues wisely, Indonesia could maintain a high growth, even when the oil-boom was over.

In this chapter, we will relate the long-run growth rate to the rate of growth of imported capital goods. The premise used here is similar to that adopted by Lee: foreign inputs are essential and efficient for domestic production. For simplicity , we assume that these foreign components are the sole source of technological progress. In other words, the only technology-enhancing factor is the imported capital goods. The growth rate of the domestic component and its share in the total physical stock will be adjusted accordingly. Due to the existence of multiple equilibra, the economy may end up in different steady-states with different long-run growth rates.

5.2 Model Specification:

In order to examine the relationship between the imported capital goods and long-run growth, we consider an economy which uses two types of capital goods, domestically produced and imported ones to make up a composite capital stock, which in turn will combine with labour to produce a simple final good for both consumption and accumulation purposes. The economy is engaged in trade with the rest of the world by importing capital goods from outside, because these capital goods are essential and more effective for the home-country. The import is possible either by borrowing or buying directly at the expense of domestic consumption and domestic accumulation. Foreign inputs are assumed, for simplicity, to be the only source of technological change and the change in productivity is affected by the rate of growth of imported capital goods. The labour force is assumed to be fixed over time, and the objective is to maximize the present value of future streams of consumption.

Formally, the main components in the model can be defined as follows:

The composite capital stock. The composite capital stock K is made up of two components: domestic K_D and imported K_M according to the following Cobb-Douglas function:

$$K = K_D^{\gamma} K_M^{1-\gamma} \qquad (5.1)$$

where K_D is the domestically produced capital stock

K_M is the imported produced capital stock

The production function and the technological progress function:

The economy in consideration produces a single good using two inputs: labour and the composite capital. The production function has the form:

$$F(K, L, t) = Ae^{zt} K^{\alpha} L^{1-\alpha} \qquad (5.2)$$

where, K is the composite capital stock, and

L is the labour force used in production

z (t) is a measure of technological progress

The change in productivity induced by the investment activity is supposed to come only from importing foreign inputs. A more general case may be considered, but since our main purpose is to consider the effects of imported capital goods on growth, this simplification is well justified. The technology is therefore, embodied in foreign machinery and equipment and is to be transferred or diffused through the imports. The extent of this transfer or diffusion may be measured by two variables: either by the ratio of the imported capital goods to the domestically produced goods (as in Lee's model) or by the growth rate of the imported capital goods

In this first case , the change in productivity is measured by:

$$\dot{w} = \frac{g(x)}{1-\alpha} - \beta w \qquad (5.3a)$$

where, similar to Jong-Wha-Lee's model, x is the ratio of the imported capital goods to the domestically produced goods, i.e. $x = \dfrac{K_M}{K_D}$

In the second case, the change in productivity induced by the flow of imported capital goods can be expressed as a function of the growth rate of the imported capital goods, that is $\dot{w} = \dfrac{g(\pi)}{1-\alpha} - \beta w$ (5.3b)

where π is the growth rate of the imported capital goods, defined by

$$\pi = \frac{\dot{K}_M}{K_M} = \frac{B}{K_M}$$

Lee (1994) uses the *ratio of imports to investment* as a factor determining the long-run growth rate. Since in his model, the domestic capital goods (or domestic new investment) are a linear function of the total capital stock, it is quite clear that his specification is equivalent to the first one. It should be noted that the first specification may be of interest in the case of a linear production function, it is no longer interesting in our model. A profit-maximising firm will tend to determine an optimal composition of two types of capital goods, based on their relative prices and their relative contributions in forming the composite capital stock. The technology function, therefore would be tied up with this choice. Therefore, in this model, we will adopt the second specification.

The imported capital flow and the budget constraint

As noted before, there are two possibilities for the economy to finance the import of capital goods. Firstly, the country can use direct financing by buying the needed equipment abroad by giving up a part of saving which would be devoted totally to building the domestic capital stock as in the case of the closed economy. Secondly, the country can import the needed equipment by *borrowing*. In this model, we consider only the second possibility. Under some conditions, the two specifications do not significantly differ from each other(*) .

Suppose that the amount of imported capital goods is B and the interest rate is r, then the budget constraint becomes

$$\dot{K}_D = f(K) - C - rK_M \qquad (5.4b)$$

$$\dot{K}_M = B.$$

The economy, as a whole, now can be described by the system of Equations (5.1) - (5.4). Following the treatment used in the previous chapters, it is better for us to deal with the transformed variables, that is with variables measured in terms of efficient unit of labour. As before, let's denote

$w(t) = \dfrac{z(t)}{1-\alpha}$ the modified technological progress function

$G(t) = \dfrac{F(t)}{e^{w(t).t}}$ total output, in terms of effective labour unit

$h_D(t) = \dfrac{k_D(t)}{e^{w(t).t}}$ the domestic capital stock in terms of effective labour unit

$c(t) = \dfrac{C(t)}{e^{w(t).t}}$ consumption in terms of effective labour unit

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(*)If the newly acquired imported capital goods is B and the relative price of those goods to the domestic goods is p , then the budget constraint has the following form:

$$\dot{K}_D = f(K) - C - pB \qquad (5.4a)$$

and $\dot{K}_M = B$

Suppose θ is the growth rate of the domestically produced capital goods, i.e. $\theta = \frac{\dot{K}_D}{K_D}$. Then the economy can be described by the following system of equations:

$$\dot{w} = \frac{g(\pi)}{1-\alpha} - \beta w \qquad (5.5)$$

$$\dot{h}_D = \theta h_D - h_D(w + \dot{w}t) \qquad (5.6)$$

$$\dot{x} = x(\pi - \theta) \qquad (5.7)$$

$$G = Ah_D^{\alpha} x^{\alpha(1-\gamma)} \qquad (5.8)$$

$$c = Ah_D^{\alpha} x^{\alpha(1-\gamma)} - h_D(\theta + rx) \qquad (5.9)$$

Equation (5.5) relates technological change to the growth rate of imported capital stock. Similar to what have been said in the previous chapters, this equation relates the gains in productivity with the learning ability of the economy by investing in foreign inputs. The parameter β represents the degree of imperfection of the transmission of knowledge over time. Equation (5.6) describes the dynamics of the effective domestic capital stock under the influence of the growth rate of physical capital and the change in technology. Equation (5.7) shows the change in composition of domestically produced capital goods and imported ones, depending on the differences between the growth rates of these two types of capital. If the domestic capital grows faster, then the total capital stock will eventually be dominated only by the domestic component. Otherwise, the foreign inputs will dominate. Along a balanced growth path, however, the two capital stocks grow at the same rate, so the composition of the total capital will be at a steady-state. Equation (5.8) represents a modified production function, where the effective output depends on the effective domestic capital stock and the relative size of imported to domestic capital. Finally, Equation (5.9) shows the relationship between consumption and others important factors such as the effective domestic capital, the ratio of foreign to domestic capital stock, the growth rate of capital and the interest rate. Derivations of these equations from the original system of Equations (5.1) - (5.4) are given in Appendix 5.A

We now turn our attention to considering the existence of a balanced growth path for the economy. Recall from Chapter 3 that a capital accumulation path is characterized by the growth rate of the capital stock. The path is feasible if it is accompanied with a non-negative stream of future consumption. Because the economy in consideration uses two types of capital for production, its accumulation path is characterized by the growth rates of both types of capital, i.e by a pair *(π, θ)* where θ is the growth rate of the domestic capital stock and π is the growth rate of the imported stock. The following Lemma specifies necessary conditions for a feasible accumulation path in this case.

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Lemma 5.1: Necessary conditions for a feasible growth path

For a capital accumulation path characterized by a pair (π, θ) to be feasible, it is necessary that

$$a)\ \frac{g(\pi)}{(1-\alpha)\beta} \geq \pi . \qquad (5.10)$$

Furthermore,

$$b)\ \text{if } \pi < \theta \text{ then } \ g(\theta) \geq (1-\alpha)\beta\theta \qquad (5.11)$$

It is worthwhile to note that these necessary conditions are similar to that in the case of a single capital good considered in Chapter 3. Condition a) states that for the economy to grow at a sustainable rate, the change in productivity induced by the imported capital goods should not be less than the growth rate of the imported capital good. Recall that a similar condition determines the region of feasible growth paths in Chapter 3. Condition b) states that if the domestic capital grows at a higher rate than the imported capital, then for the path to be feasible, both rates of growth must be in the region determined by the Condition a). Except for this case , there is no special condition to impose on the growth rate of the domestic capital. This property is different from the one discussed in the case of a closed economy and will have important implications for the optimality conditions which will be considered in the next section.

Proof : From the budget constraint (5.4b) and following a treatment in Chapter 3, section 3.3, it is clear that, an accumulation path is feasible if

$$(1-\alpha)w+\gamma\alpha\theta+(1-\gamma)\alpha\pi \geq \max(\pi,\theta) \qquad (5.12)$$

The left-hand side of the above inequality is the induced growth rate of the output under the conditions that the domestic capital goods grow at a rate equal to θ and the imported capital goods grow at a rate equal to π. This growth rate of output consists of two components: one is due to change in technology $(1-\alpha).w$ and the other is the growth rate of the composite capital stock as weighted sum of the rates of growth of two components: domestic and foreign (second and third term). On the other hand, since the growth rate of productivity induced by the flow of imported capital goods is defined by Equation (5.5) , the function $w(t)$ can be expressed explicitly as follows:

$$w(t) = \frac{g(\pi)}{(1-\alpha)\beta}\left(1 - e^{-\beta t}\right)$$

It follows immediately that $w < \dfrac{g(\pi)}{(1-\alpha)\beta}$

Denote the left-hand side of (5-12) by $LHS = (1-\alpha)w + \gamma\alpha\theta + (1-\gamma)\alpha\pi$, then

$$LHS \le \frac{g(\pi)}{\beta} + \gamma\alpha\theta + (1-\gamma)\alpha\pi \text{ for all possible } (\pi, \theta) \tag{5.13}$$

Suppose by contrast, that a pair (π, θ) is feasible but Condition a) does not hold. That is $\dfrac{g(\pi)}{\beta} < (1-\alpha)\pi$. It becomes clear now that:

If $\theta \le \pi$, then from (5.13) it follows

$$LHS \le \frac{g(\pi)}{\beta} + \gamma\alpha\theta + (1-\gamma)\alpha\pi < (1-\alpha)\pi + \gamma\alpha\pi + (1-\gamma)\alpha\pi = \pi$$

On the other hand , if $\pi < \theta$, then

$$LHS \le \frac{g(\pi)}{\beta} + \gamma\alpha\theta + (1-\gamma)\alpha\pi < \frac{g(\pi)}{\beta} + \alpha\theta < (1-\alpha)\pi + \alpha\theta < \theta \tag{5.14}$$

In both cases, the feasibility condition (5.12) does not hold. Therefore, the accumulation path is not feasible.

Condition b) is not difficult to prove. In fact, if $\pi < \theta$, then as has been shown in (5.14)

$LHS < \frac{g(\pi)}{\beta} + \alpha\theta < \frac{g(\theta)}{\beta} + \alpha\theta$ since the function $g(\pi)$ is monotonicly increasing in π. Therefore, the path is not feasible if $g(\theta) < (1-\alpha)\beta\theta$ ■ *QED*

Lemma 5.1 specifies necessary conditions for a feasible growth path. It also provides the sufficient condition for a feasible growth path. In fact, similar to what has been considered in Chapter 3, the existence of a perpetual growth path, in the case of an open economy depends on the existence of a positive solution to the inequality (5-10) -(5-11). It can be shown that an accumulation path characterized by a pair (π, θ) satisfying inequalities (5-10)-(5-11) is , in fact a perpetual path. Furthermore, if the pair (π, θ) satisfy conditions (5-10)-(5-11) as equalities, then the corresponding accumulation path represents a balanced growth path, since the output, the domestic capital stock and the foreign capital stock all grow at the same rate.

5.3 Optimal Balanced Growth Path

We now consider a standard intertemporal optimal choice facing a representative household in the economy described in the previous section. The household seeks to maximize his/her overall utility from the steams of future consumption with a constant rate of time preference ρ.

$$U = \int_0^\infty e^{-\rho t} u(C_t) dt = \int_0^\infty e^{-\rho t} \ln(C_t) dt$$

where $\rho > 0$ is the constant rate of time preference .

The economy is described by the system of equations (5-1)- (5-4) or in terms of transformed variables by the system (5-5)-(5-9). For an easy exposition, a new variable $v = wt$ is introduced , instead of variable w. The new variable represents the accumulated gains (or increase) in the productivity due to the import of capital goods. In terms of new transformed variables, the utility from consumption can be expressed as follows:

$$u(C_t) = \ln(C_t) = wt + \ln(c_t) =$$

$$= v + \ln(G - h_D(\theta + rx))$$

Following the remark made in the Lemma 5.1, we can restrict our consideration to only feasible growth paths. Therefore, it is possible and useful to include an additional restriction to the whole system. The intertemporal optimization problem now can be written as the following

$$\max \int_0^\infty e^{-\rho t}(v + \ln(G - h_D(\theta + rx)dt \qquad (5.15)$$

subject to

$$\dot{h}_D = h_D(\theta - \dot{v}) \qquad (5.16)$$

$$\dot{v} = \frac{g(\pi)t}{1-\alpha} - \beta v + \frac{v}{t} \qquad (5.17)$$

$$\dot{x} = x(\pi - \theta) \qquad (5.18)$$

$$G = Ah_D^\alpha x^{\alpha(1-\gamma)} \qquad (5.19)$$

$$c = Ah_D^\alpha x^{\alpha(1-\gamma)} - h_D(\theta + rx) \qquad (5.20)$$

$$g(\pi) \geq (1-\alpha)\beta\pi \qquad (5.21)$$

The meanings of each equation has already been given in the previous section. Equation (5.21) specifies a condition for a feasible growth rate of the imports. It should be noted that, in the case when the technology-shock generation function $g(.)$ is S-shaped, the system can have two distinguished balanced growth rates π_1 and π_2: a result already discussed in Chapters 3 and 4. Equation (5.12) can be replaced by a simpler constraint

$$\theta_1 \leq \theta \leq \theta_2 \qquad (5.22)$$

Before proceeding to the necessary conditions for the problem (5-15)-(5-21), we assume, for simplicity, that A = 1 and recall that the partial derivatives of the production function $G(.)$ with respect to its variables h_D and x can be found as follows:

$$\frac{\partial G}{\partial h_D} = \frac{\alpha G}{h_D} = \alpha h_D^{\alpha-1} x^{\alpha(1-\gamma)} \qquad (5.23)$$

and $$\frac{\partial G}{\partial x} = \frac{\alpha(1-\gamma)G}{x} = \alpha(1-\gamma) h_D^{\alpha} x^{\alpha(1-\gamma)-1} \qquad (5.24)$$

The Hamiltonian is given by:

$$H = v + \ln(G - h_D(\theta + rx)) + \mu\left(\frac{g(\pi)t}{1-\alpha} - \beta v + \frac{v}{t}\right) + \lambda h\left(\theta - \frac{g(\pi)t}{1-\alpha} + \beta v - \frac{v}{t}\right) + \eta x(\pi -$$
(5.25)

Applying the standard dynamic programming technique to this problem gives us the following FOCs:

$$\frac{\partial H}{\partial \theta} = -\frac{h_D}{G - h_D(\theta + rx)} + \lambda h_D - \eta x = 0 \qquad (5.26)$$

$$\frac{\partial H}{\partial \pi} = \frac{g'(\pi)}{h(1-\alpha)}(\mu - \lambda h_D) + \eta x \qquad (5.27)$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial v} = \rho\mu + (\mu - \lambda h_D)(\beta - \frac{1}{t}) - 1 \qquad (5.28)$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial h_D} = \rho\lambda - \frac{1}{G - h_D(\theta + rx)}\left[\frac{\partial G}{\partial h_D} - (\theta + rx)\right] - \lambda(\theta - \dot{v})$$

(5.29)

$$\dot{\eta} = \rho\eta - \frac{\partial H}{\partial x} = \rho\eta - \frac{1}{G - h_D(\theta + rx)}\left[\frac{\partial G}{\partial x} - rh_D\right] - \eta(\pi - \theta)$$

(5.30)

To verify that the higher balanced growth path satisfies FOCs, we apply the same procedures as in Chapter 4 as follows.

Let's consider a balanced growth path which is, as before, characterized by a stationary growth rate of both imported and domestically produced capital goods at their upper or lower bounds , that is

$\theta = \pi$ and $\dfrac{g(\pi)}{1-\alpha} = \beta\pi$

Given this value of the growth rate in capital , the new productivity variable v can be found explicitly by solving the differential Equation (4.15). It yields a simple functional form

$$v(t) = \frac{g(\pi)t}{(1-\alpha)\beta}$$

Along this accumulation path, the effective capital stock h and the ratio of imported capital goods to the domestically produced capital x remain unchanged, hence attain their steady-state. Thus

$h(t) = h^*$ and $x = x^*$ for all t

Let (h^*, x^*) are chosen so that the following condition holds:

$$\frac{\gamma\alpha G}{h} = \theta + \rho \qquad (5.31)$$

The right-hand side of Equation (5.31) is the effective discount rate in the Barro and Sala-i-Martin interpretation and have been used in Chapter 4. The left-hand side expression is the marginal productivity of the domestically produced capital. Therefore, the above specified condition simply states that: at the equilibrium the marginal productivity of domestic capital should be equal to the effective discount rate.

The steady-state values of the co-state variables λ and η are given by

$$\overline{\lambda} = \frac{(1-\gamma)\theta - \gamma rx + \rho}{\rho\gamma F} \qquad (5.32)$$

and $\overline{\eta} = \dfrac{h((1-\gamma)(\theta+\rho) - rx)}{\rho\gamma Fx}$. $\qquad (5.33)$

where $F = G - h_D(\theta + rx)$

Since λ and η are the modified shadow prices of the domestic and foreign capital goods respectively, they must be non-negative. This implies the following condition for the steady value of the ratio of foreign to domestic capital goods:

$x \le \dfrac{(1-\gamma)\theta + \rho}{r\gamma}$ for λ to be non-negative and

$x \le \dfrac{(1-\gamma)(\theta + \rho)}{r}$ for η to be non-negative.

Given these fixed values of the co-state variable λ and the effective domestic capital stock h_D, the time path of the co-state variable μ associated with the balanced growth path can be found by solving the differential Equation (5.28). It should be noted that this differential Equation has exactly the same form as in Equation (4.20) in Chapter 4. The solution is given by:

$$\mu(t) = \frac{Me^{(\rho+\beta)t}}{t} + \frac{1-\rho\lambda h_D}{\rho+\beta} + \frac{1-\rho\lambda h_D}{(\rho+\beta)^2 t} + \lambda h_D \tag{5.34}$$

where M is a constant.

As in Chapter 4, it can be seen that the higher growth path satisfies the necessary optimal conditions. Applying the same procedure used in Chapter 4 we can come to a similar conclusion that one of the two balanced growth paths is optimal for the dynamic optimization problem (5.15)- (5.21)

The choice between two balanced growth paths, once again is a matter of dealing with the trade-off between the present and the future consumption. The utility generated by moving along one of the two balanced growth paths can be written as follows:

$$U_{\max} = \int_0^\infty e^{-\rho t}(\theta t + \ln h_D + \ln \frac{\theta + \rho - \alpha\gamma(\theta + rx)}{\alpha\gamma})dt \tag{5.35}$$

The utility, therefore depends on both the growth rate θ and the level of consumption measured by the last two terms in Formula (5.35). Shifting from the lower growth path to the higher one will increase the growth rate but at the same time lower the level of consumption. This kind of trade-off between the growth effect and the level effect will determine which balanced growth path would be optimal: an impatient society tends to take the lower growth path, while a more forward-looking one may be better off by choosing the higher one.

5.4 Trade Distortions, Capital Control and Growth.

Since, in this model, there are two state variables h_D and x which jointly determine the marginal productivity of domestic capital, it is useful to consider these optimal values jointly with the corresponding optimal growth rate. The relationship between these three variables in determining the long-run optimal growth path is depicted in Figure 5.3.

In this Figure, the horizontal axis represents the ratio of imported to the domestic capital goods x and the vertical axis represents the effective capital stock h. Given the production function in (5), the marginal productivity of domestic capital can be expressed as follows

$$\frac{\partial G}{\partial h_D} = \frac{\alpha G}{h_D} = \alpha h_D^{\alpha-1} x^{\alpha(1-\gamma)}$$

Consider now an iso-productivity curve in the *(x, h)* space which consists of all possible combinations of x and h that yield the same value for the marginal productivity of domestic capital, i.e

$$\frac{\partial G}{\partial h_D} = \alpha h_D^{\alpha-1} x^{\alpha(1-\gamma)} = const \tag{5.36}$$

It can be seen that the iso-productivity curve is upward-sloping in the *(x, h)* space. It may be concave or convex to the origin depending on the relative values of the two parameters α and γ. In fact, taking the total differential of (5.36) gives us

$$\alpha(\alpha-1)h^{\alpha-2}x^{\alpha(1-\gamma)}dh+\alpha^2(1-\gamma)h^{\alpha-1}x^{\alpha(1-\gamma)-1}dx=0 \qquad (5.37)$$

Since $0 < \alpha, \gamma < 1$, it follows that $\frac{dh}{dx} > 0$, meaning that the iso-productivity curve is upward-sloping.

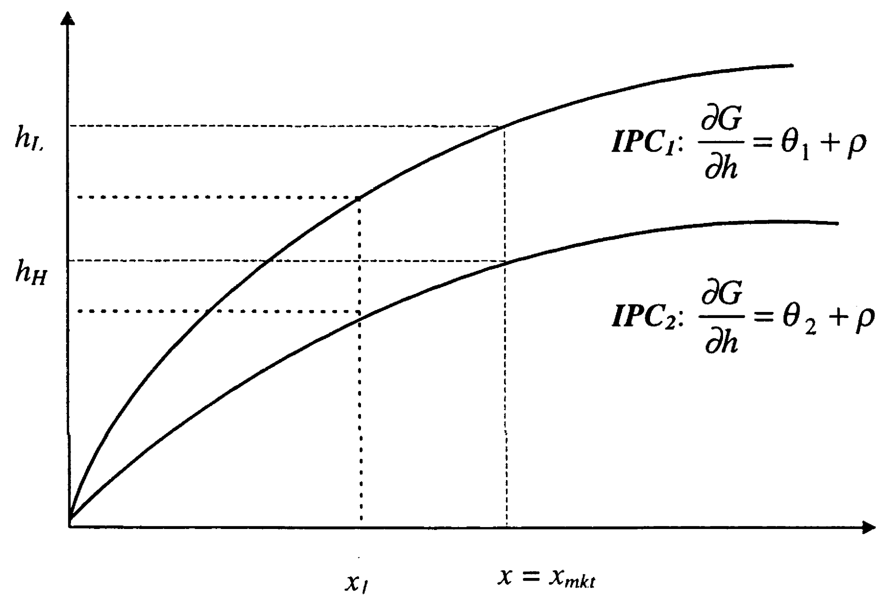
It is worth noting that due to the convexity of the production function or the diminishing marginal productivity, an increase in the constant term in Equation (5.36) i.e an increase in the marginal productivity of capital will cause the corresponding curve to shift *down* . This is because if the ratio of imported to domestic capital goods is fixed, an increase in the marginal productivity is possible only with a smaller effective capital stock .

The above primary analysis now can be used to examine the relationship between the optimal effective capital stock and the long-run growth rate. The equilibrium conditions in the capital market require that the marginal productivity of domestic capital should equal the effective discount rate. Two different balanced growth paths are associated with two different effective discount rates $\theta_1 + \rho$ and $\theta_2 + \rho$. In Figure (5.1) the two iso-productivity curves associated with the two balanced growth paths are IPC_1 and IPC_2 . Suppose that the imported to domestic capital ratio is determined by the market equilibrium condition that is

$$x = x_{mkt} = \frac{1-\gamma}{r\gamma}$$

then if the economy prefers the higher growth path, the required effective capital stock will equal h_H . Otherwise, moving along the lower growth path would require a higher effective capital stock equal to h_L . Since the iso-productivity curve for the lower path is above the corresponding curve for the higher one, it is obvious that $h_L > h_H$. Therefore, there is a trade-off between the growth rate and the current level of consumption: the higher growth path is accompanied by the lower level of consumption and *vice versa*. As in the case of a closed economy considered in Chapter 4, time-preference will play an important role in which balanced growth path is to be chosen.

Figure 5.3 Trade Distortions, Capital Control and Growth.



The possible effects of trade distortions and capital control on growth can be analysed, using the above paradigm.

As is well known, one of the first direct effects of any trade distortion and capital control is to restrain the flow of imported capital goods into the country under consideration. A tax imposed on imports, for example, makes the imported capital goods relatively more expensive and forces the producer to move away from this imported source. As a result, the ratio of imported to domestically produced capital stocks tend to decline. In Figure 5.3 this means that the market determined ratio falls from x to x_l. In order to maintain the domestic capital market in equilibrium along the designed growth path, the effective domestic capital stock should also so decrease. There are two possibilities for the entire economy: either to adopt a higher growth path with a lower level of capital, hence, a lower level of actual consumption; or to accept a lower growth path with higher current consumption. Suppose initially the economy is already at the higher path, the new choice may result in a shift in policy: from the higher path to the lower one. Therefore, a restricted trade policy which limits the availablility of foreign inputs to the economy has potential damaging effects not only on the welfare and short-run growth but also on the long-run growth performance of the country.

CHAPTER 6

CONCLUSION

New growth models which emerged less than a decade ago, show no sign of fading. On the contrary, this field of research represents one of the most widely debated subjects in economic theory, at the moment and may do so for many years to come. Emphasis has been placed on the role of technological progress in growth and factors determining technological change. Modelling technological change, therefore, is one of the most interesting and challenging topics in growth theory. Given the existence of a vast body of literature on growth models and different approaches to this complicated problem, the research carried out here has had a modest, but nevertheless important objective: to show the possibility of an endogenous growth model based on the learning effects via the investment activity.

It can be said that the model constructed here is functionally equivalent to some of the previous endogenous models, such as those of Romer and Lucas. The difference is which factor is considered as technology-enhancing and how the factor may affect technological change. While in the Lucas model, the emphasis is placed on human capital, and there is no explicit mention about technological change, it can be easily shown that functionally the Lucas model can be reduced to a model with human capital being a technology-enhancing factor, and the increase in productivity is a linear function of the rate of growth of the human capital. This can be done by the following:

Suppose the technology in the Lucas production function is labour-embodied, so

$$A(t) = h(t)^{1-\alpha}$$

In terms of growth rates , these two variables, the technological progress and the human capital stock are related by the following equation

$$\frac{\dot{A}}{A} = (1-\alpha)\frac{\dot{h}}{h}$$

Thus, the Lucas model is reduced to a functionally equivalent model when the technological progress is explicitly included and is affected by the human capital accumulation rate via an educational process.

The Romer model is closer to the model in this thesis. The difference has been already mentioned in Chapter 3. It resides in the fact that, in the Romer model, the physical capital stock is fixed and only knowledge is allowed to vary. The technological progress is a function of the rate of growth of knowledge. In my approach, the evolution of the physical capital stock is explicitly considered and knowledge can be seen as implicitly embodied in the technological progress function.

Another modelling feature of this model is that it can provide some responses to Solow's objection about ' the easy endogenous growth', a notion that Solow has used to argue that a growth model can easily become endogenous if we adopt the assumption that an innovation generates a proportionate rather than an absolute increase in the total factor productivity(*) . This concerns the adoption of a linear

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(*) Solow (1994, p53) puts it:

'It is easy to agree that the flow of innovations per unit time depends on the amount of resources devoted to creating them. If an innovation generates a proportionate increase in A, function of knowledge or human capital or even physical capital creation in some endogenous growth models. More generally, the Solow objection concerns the following question. Which is more appropriate to assume the relationship between a technology-enhancing factor (such as knowledge or human capital) and technological change in terms of *growth-to-growth* or *level-to-level* linkages? However, the example of using a modified vintage theory to derive a hypothetical technological progress function in Chapter 3 shows that there is a well justified way to use *both* these linkages in one integrated specification: the growth-to-growth approach does not exclude, but in fact is based on the level-to-level specification.

Although attention has been focused only on the role of investment (internal or external) in growth, our model is quite flexible and very open in choosing a technology-enhancing factor. This flexibility does not tie up the model to any particular type of capital; therefore it can be applied to a wide range of models. One important consideration is: which factor should be considered as technology-enhancing and how the change in technology and productivity is related to the change in this factor?

From the empirical viewpoint, this research can explain some stylised facts; in particular it can be used to explain why the growth performance of different countries and regions has been so diverse. One possible explanation for this diversity is that different countries or regions have different time preferences and/or possess

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then we have a theory of easy endogenous growth. Spend more resources on R&D, there will be more innovations per years, and the growth rate of A will be higher. But suppose that an innovation generates only an absolute increase in A: then greater allocation of resources to R&D buys a one-time jump in productivity, but not a faster rate of productivity growth.. I do not know which is the better assumption, and these are only two of many possibilities' different learning opportunities. Different levels of human capital which influence directly the learning ability will have a growth effect even in the long-run. A developing country can improve its growth performance by opening up to the rest of the world, thereby facilitating the import of more effective capital goods. Choosing the right partner with more advanced technology for a foreign direct investment joint-venture may help a less developed country not only partially to overcome the traditional savings gap and lack of investment resources, but also provide a better opportunity for sustainable long-run growth. All these 'stylised facts' and well-documented observations can find, at least, a partial explanation in this model.

One interesting and maybe not so obvious implication of the model results from the fact that the long-run growth rate is not a continuous function of some policy variables, but rather a step function. There are two opposite implications: firstly, a piecemeal policy change may be not enough for achieving a better long-run performance, and secondly, a small change in some restriction policy may lead to a dramatic decline in the long-run growth rate. All depend on how far the actual value of this or that policy variable (such as saving rate, trade restrictions, etc.) is from the threshold one. A policy aimed at increasing the national saving ratio from 15% to 25%, for example, represents a great effort by a developed country, but maybe not enough to foster the country growth to a level which is enjoyed by some newly-industrialised countries. By contrast, a small change in trade policies in an already highly protected country may result in a significant drop in the long-run growth rate. This suggests a prudent approach in policy designing.

Finally, the research undertaken in this thesis is by no means complete. Rather it represents only a small step in understanding the linkages between the inside economic activity and the long-run growth performance through the investment and learning process. Empirical work to test some implications of the model, especially the relationship between the growth rate of imported capital goods and long-run growth, similar to the Lee model, is desirable. It would be worthwhile research for the future. Nevertheless the thesis contributes to the development of an analytical framework that underpins the belief that it may be possible for economies to achieve long-run growth and prosperity.

APPENDIX 4.A

Consider the following problem

$$\max \int_0^{\infty} e^{-\rho t}(wt + \ln c)dt \qquad (1)$$

$$\dot{w} = \frac{g\left(h^{\alpha-1} - ch^{-1}\right)}{1-\alpha} - \beta w \qquad (2)$$

$$\dot{h} = h^{\alpha} - c - h(w + \dot{w}t) \qquad (3)$$

The Hamiltonian is given by

$$H = wt + c + \lambda\left(\frac{g(h^{\alpha-1} - ch^{-1})}{1-\alpha} - \beta w\right) + \mu(h^{\alpha} - c - h(w + t\left(\frac{g(h^{\alpha-1} - ch^{-1})}{1-\alpha} - \beta w\right)$$

Denote $g(h^{\alpha-1} - ch^{-1}) = g(.)$ and $g'(h^{\alpha-1} - ch^{-1}) = g'(.)$. The partial derivatives of function H with respect to its variables *c*, *h* and *w* can be expressed as follows:

$$\frac{\partial H}{\partial c} = \frac{1}{c} - \mu - \frac{g'(.)}{h(1-\alpha)}(\lambda - \mu ht) \qquad (4)$$

$$\frac{\partial H}{\partial w} = t - \mu h - \beta(\lambda - \mu ht) \qquad (5)$$

$$\frac{\partial H}{\partial h} = \frac{g'(.)}{1-\alpha}(\lambda - \mu ht)\left[(\alpha - 1)h^{\alpha-2} + ch^{-2}\right] + \mu\left[\alpha h^{\alpha-1} - w - \frac{tg(.)}{1-\alpha} + \beta wt\right] \qquad (6)$$

The first-order necessary conditions (FOCs) for this problem are as follows:

$$\frac{\partial H}{\partial c} = 0 \qquad (7)$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial h} \qquad (8)$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial w} \qquad (9)$$

To find an optimal solution, I adopt the following procedure (like in the Lucas approach).

1. Guess a candidate for optimal solution. (a steady-state is a good one)
2. Verify FOC , by choosing an appropriate parameter for this candidate.

Suppose *(h, w)* is a steady-state value, and c is correspondent consumption at this point.

Then $\dot{h} = \dot{w} = 0$ implies:

$$\beta w = \frac{g(.)}{1-\alpha} \text{ and } w = h^{\alpha-1} - ch^{-1} \qquad (9)$$

Denote $A = \dfrac{g'(.)}{(1-\alpha)h}$

Taking into account (13), Equations (4)-(6) which reflect the actual values of partial derivatives at steady-state point, can be rewritten as follows:

$$\frac{\partial H}{\partial c} = \frac{1}{c} - \mu - A(\lambda - \mu ht) \qquad (4a)$$

$$\frac{\partial H}{\partial w} = t - \mu h - \beta(\lambda - \mu ht) \qquad (5a)$$

$$\begin{aligned}\frac{\partial H}{\partial h} &= A(\lambda - \mu ht)\left[(\alpha-1)h^{\alpha-1} + ch^{-1}\right] + \mu(\alpha h^{\alpha-1} - w) = \\ &= \frac{(\alpha-1)h^{\alpha-1} - ch^{-1}}{c} = \frac{B}{c}\end{aligned} \qquad (6a)$$

where $B = (\alpha-1)h^{\alpha-1} - ch^{-1}$

The next step is to find an appropriate quintuple *(c, h, w, μ, λ)* which satisfies Equations (2), (4), (5) and (6). If this quintuple also satisfies Equation (8), then it is an optimal solution to the optimisation problem.

It should be noted that the triple *(c, h, w)* defined by Equation (13) already satisfies Equations (5) and (6). The following step is to find corresponding time-function μ and λ associated with this triple

Combining (11) and (9a), we have

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial h} = \rho\mu - \frac{B}{c} \qquad (11a)$$

This differential equation with respect to μ has a particular (stationary) solution, given by

$$\mu = \frac{B}{c\rho} \qquad (14)$$

Given above specified values of *c, h, w* and μ, the time-function λ can be found from Equation (7a) and (10), namely

$$\lambda(t) = \mu ht + \frac{1}{A}\left[\frac{1}{c} - \mu\right] = \mu ht + \frac{1}{Ac}\left[1 - \frac{B}{\rho}\right] \quad (15)$$

On the other hand, given fixed values of μ and *h*, Equations (8a) and (12) leads to a differential equation in λ

$$\begin{aligned}\dot{\lambda} &= \rho\lambda - \frac{\partial H}{\partial w} = \rho\lambda - t + \mu h + \beta\,(\lambda - \mu ht) = \\ &= (\rho + \beta\,)\lambda - (1 + \beta\mu h)t + \mu h\end{aligned} \qquad (12a)$$

The solution to this equation is

$$\lambda(t) = \frac{1 + \beta\mu h}{\rho + \beta}t + \frac{1 - \rho\mu h}{(\rho + \beta\,)^2} \quad (16)$$

It is clear now that for the quintuple *(c, h, w,* μ*,* λ*)* satisfy all FOCs, two expressions for $\lambda(t)$ in (15) and (16) must coincide. This imposes the following conditions for *h* and *c* at the steady state:

$$\begin{cases} \rho\mu h = 1 \\ B = \rho \end{cases}$$

However , the above system of equations has no solution. This mean that the steady-states in the usual sense are not optimal . It raises the possibility that the optimal solution may be in the form:

$$h_{opt} = h^* + h(t) \quad \text{and} \quad \mathrm{w}_{opt} = w^* + w(t)$$

where h_{opt} and w_{opt} are the optimal solutions and,

h^* and w^* are the steady-states

$$\lim_{t\to\infty} h(t) = \lim_{t\to\infty} w(t) = 0$$

Appendix 4.B

$$\dot{\mu} = \rho\mu + (\mu - \lambda h)(\beta - \frac{1}{t}) - 1$$

Let $m = \mu - \lambda h$. Then

$$\dot{m} = \dot{\mu} = \rho\mu + (\mu - \lambda h)(\beta - \frac{1}{t}) - 1 = m(\rho + \beta - \frac{1}{t}) - 1 + \rho\lambda h$$

Let $I(t) = e^{-\int(\rho+\beta-\frac{1}{t})dt} = te^{-(\rho+\beta)t}$

$$mte^{-(\rho+\beta)t} = (-1 + \rho\lambda h)\int te^{-(\rho+\beta)t}dt$$

$$= \frac{1-\rho\lambda h}{\rho+\beta}\left[te^{-(\rho+\beta)t} + \frac{1}{\rho+\beta}e^{-(\rho+\beta)t}\right] + M$$

where M is a constant.

Therefore

$$m(t) = \frac{Me^{(\rho+\beta)t}}{t} + \frac{1-\rho\lambda h}{\rho+\beta} + \frac{1-\rho\lambda h}{(\rho+\beta)^2 t}$$

It implies the following explicit solution for function μ

$$\mu(t) = \frac{Me^{(\rho+\beta)t}}{t} + \frac{1-\rho\lambda h}{\rho+\beta} + \frac{1-\rho\lambda h}{(\rho+\beta)^2 t} + \lambda h$$

Appendix 5.A

Derivation of main equations:

Equation (5.6):

By definition, $h_D = e^{-(1-\alpha)w} K_D$. It follows:

$$\dot{h}_D = (-w - \dot{w}t)e^{-wt} K_D + e^{-wt} \dot{K}_D$$

$$= e^{-wt} K_D \left(-w - \dot{w}t + \frac{\dot{K}_D}{K_D} \right) = h_D(\theta - w - \dot{w}t)$$

Equation (5.7)

The ratio of imported capital goods to the domestically produced capital is defined by

$x = \dfrac{K_M}{K_D}$. It follows: $\ln(x) = \ln(K_M) - \ln(K_D)$

Taking derivatives of both sides of this expression with respect to *t* yields the following.

$\dfrac{\dot{x}}{x} = \dfrac{\dot{K}_M}{K_M} - \dfrac{\dot{K}_D}{K_D}$. Therefore $\dot{x} = x(\pi - \theta)$

Equation (5.9).

From the budget constraint :

$$\dot{K}_D = f(K) - C - rK_M = Ae^{(1-\alpha)wt} K_D^{\gamma\alpha} K_M^{(1-\gamma)\alpha} - C - rK_M$$

Dividing both sides by K_D give :

$$\theta = \frac{Ae^{(1-\alpha)wt}}{K_D^{1-\alpha}} x^{\alpha(1-\gamma)} - \frac{C}{K_D} - rx$$

Therefore $\theta = Ah_D^{\alpha-1} x^{\alpha(1-\gamma)} - \dfrac{c}{h_D} - rx$

It follows: $c = Ah_D^{\alpha} x^{\alpha(1-\gamma)} - h_D(\theta + rx)$

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